SeedTree: A Dynamically Optimal And Local Self-Adjusting Tree

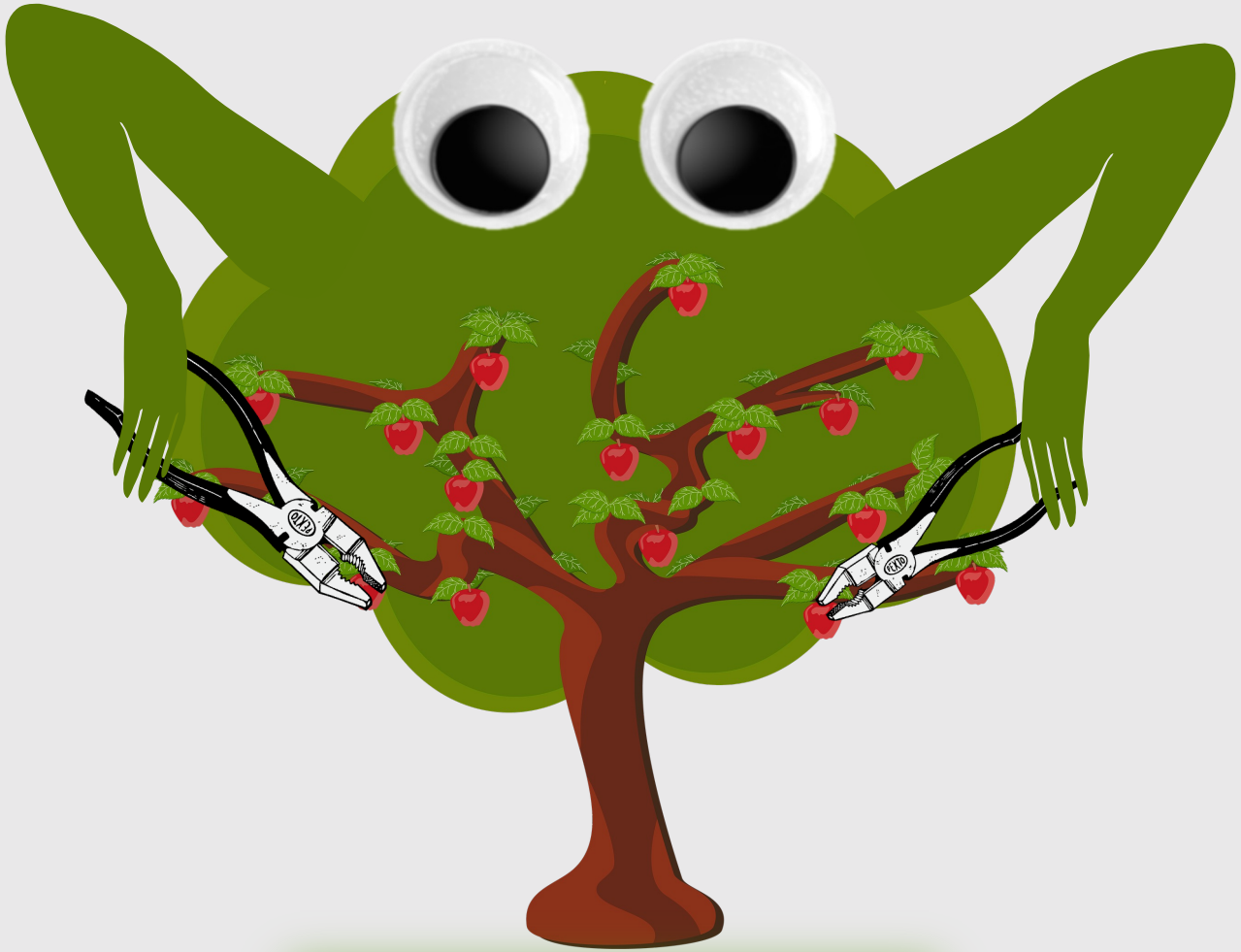
Arash Pourdamghani

Joint work with Chen Avin, Robert Sama, Stefan Schmid

Dutch Optimization Seminar

29 June 2023

An adopted version of INFOCOM'23 Talk



SeedTree at a glance

SeedTree

1) Why?

2) What?

3) How?

SeedTree

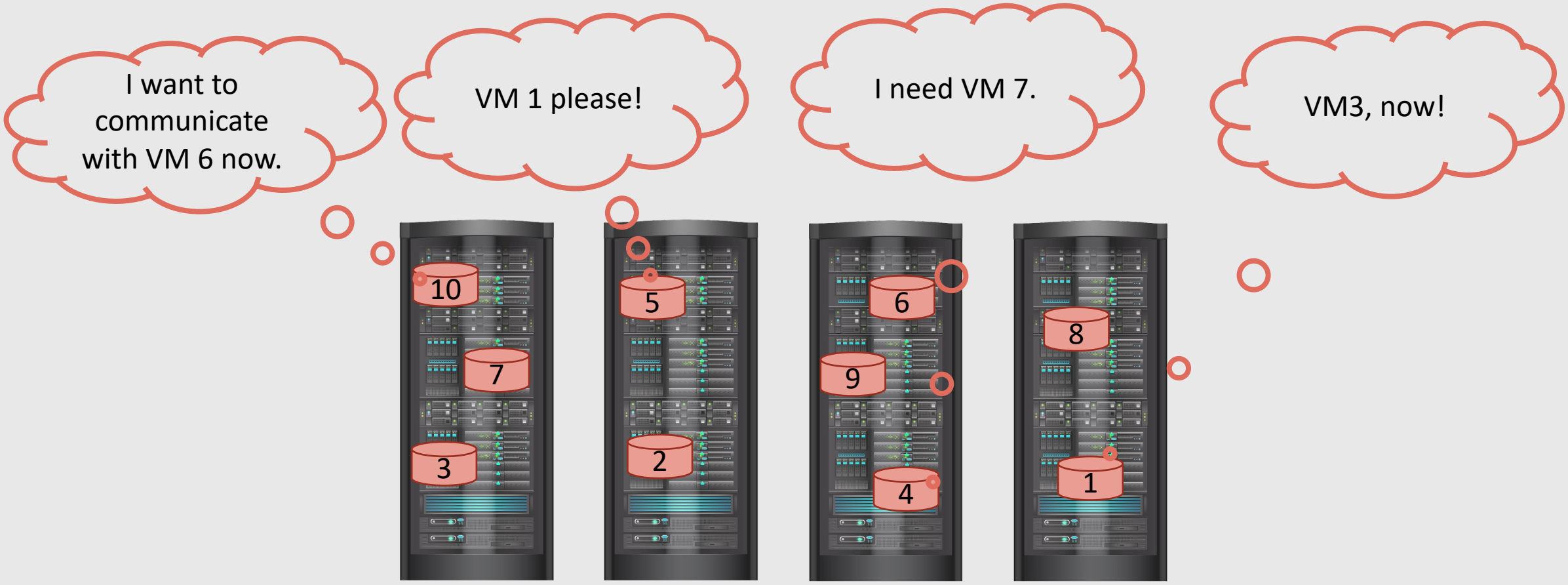
1) Why?

2) What?

3) How?

Servers and VMs

Online Request Sequence



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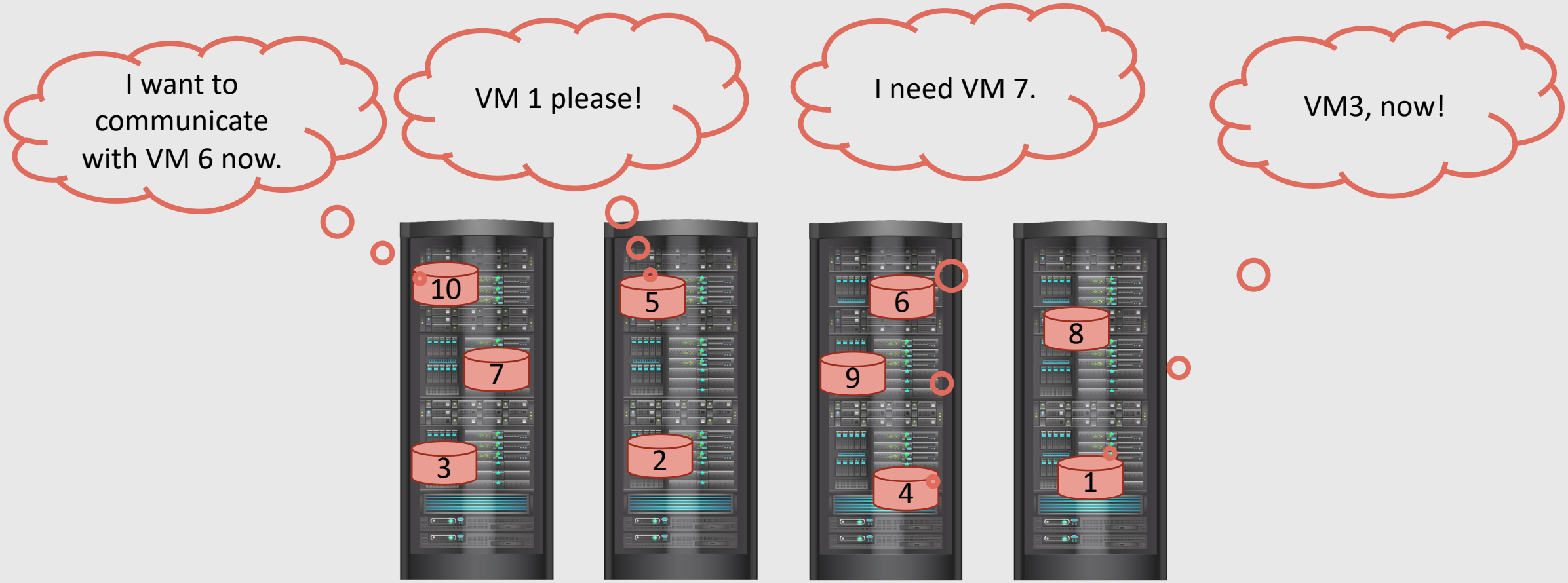
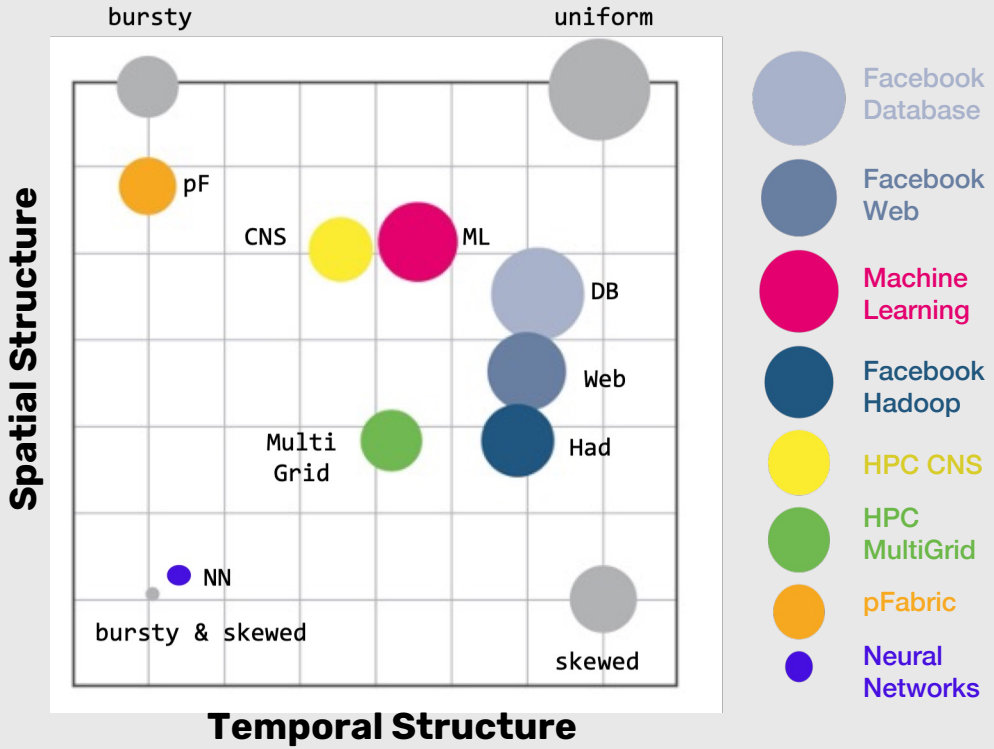


Do we have any structure in the demand?

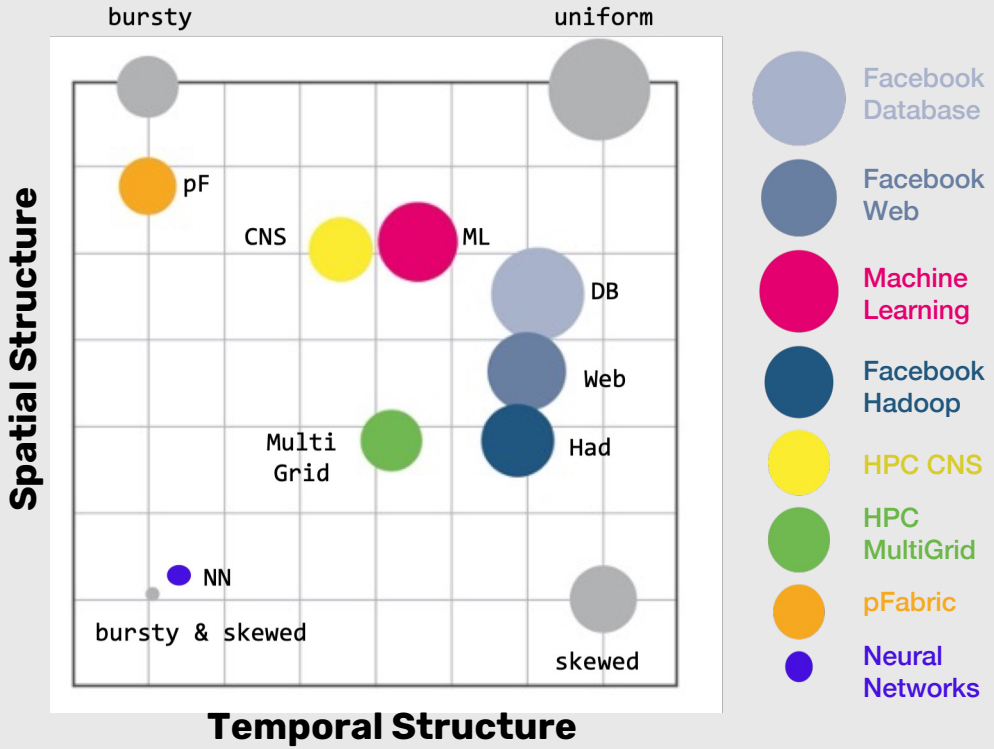
Structure in The Demand



On the Complexity of Traffic Traces and Implications
Avin et al., ACM SIGMETRICS 2020

Structure in The Demand

Can we design a self-adjusting network that utilizes demand?



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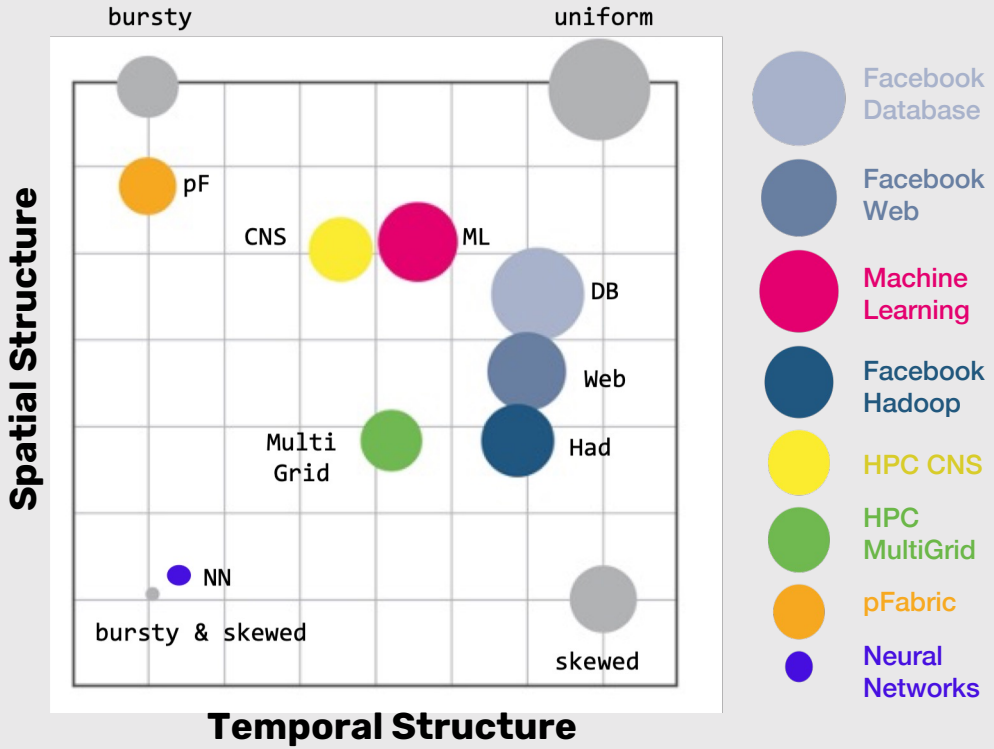
Structure in The Demand

Can we design a self-adjusting network that utilizes demand?

Let us start by a dynamically optimal self-adjusting tree!



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SeedTree

1) Why?

2) What?

3) How?

Requests From A Single Source

I want to communicate with VM 6 now.

VM 1 Please!

I Need VM 7.

VM3, now!

Using Local Routing (i.e., Without A Routing Table)!

Network destination	Netmask	Gateway	Interface	Metric
0.0.0.0	0.0.0.0	192.168.0.1	192.168.0.100	10
127.0.0.0	255.0.0.0		127.0.0.1	1
192.168.0.0	255.255.255.0		168.0.100	10
192.168.0.100	255.255.255.255	127.0.0.1	127.0.0.1	10

Considering A Binary Tree Structure on Servers

Allowing Item Movements on Edges

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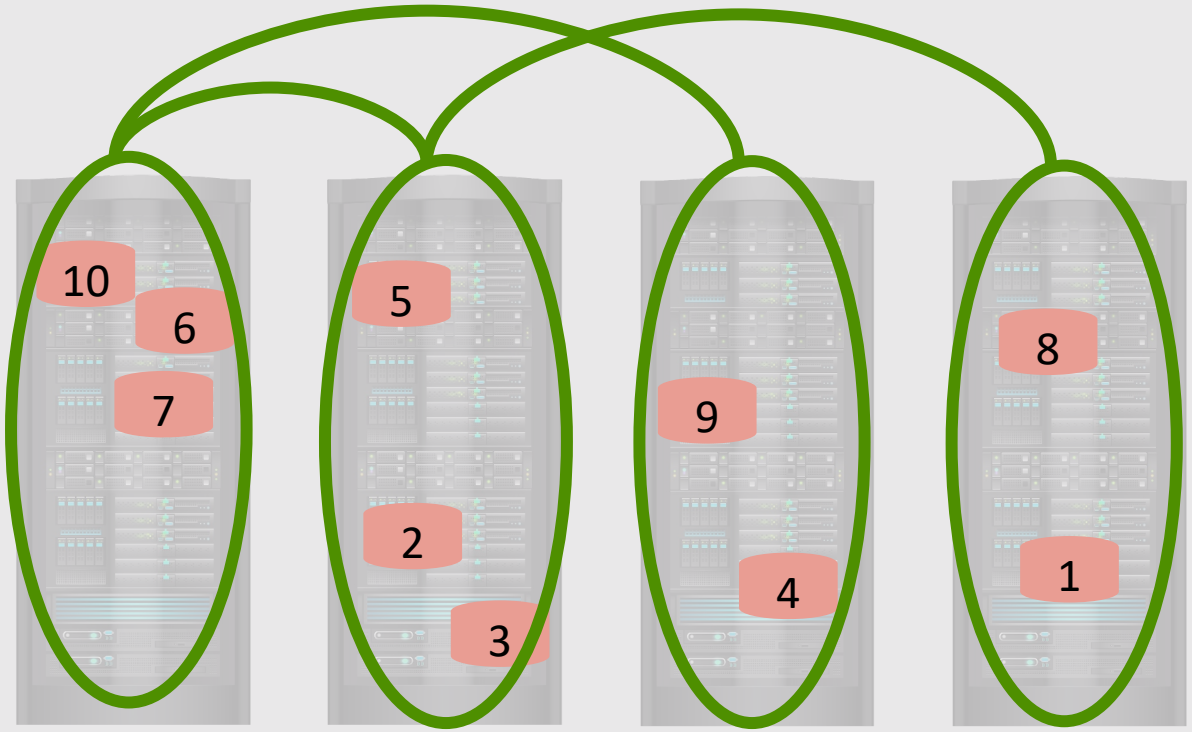
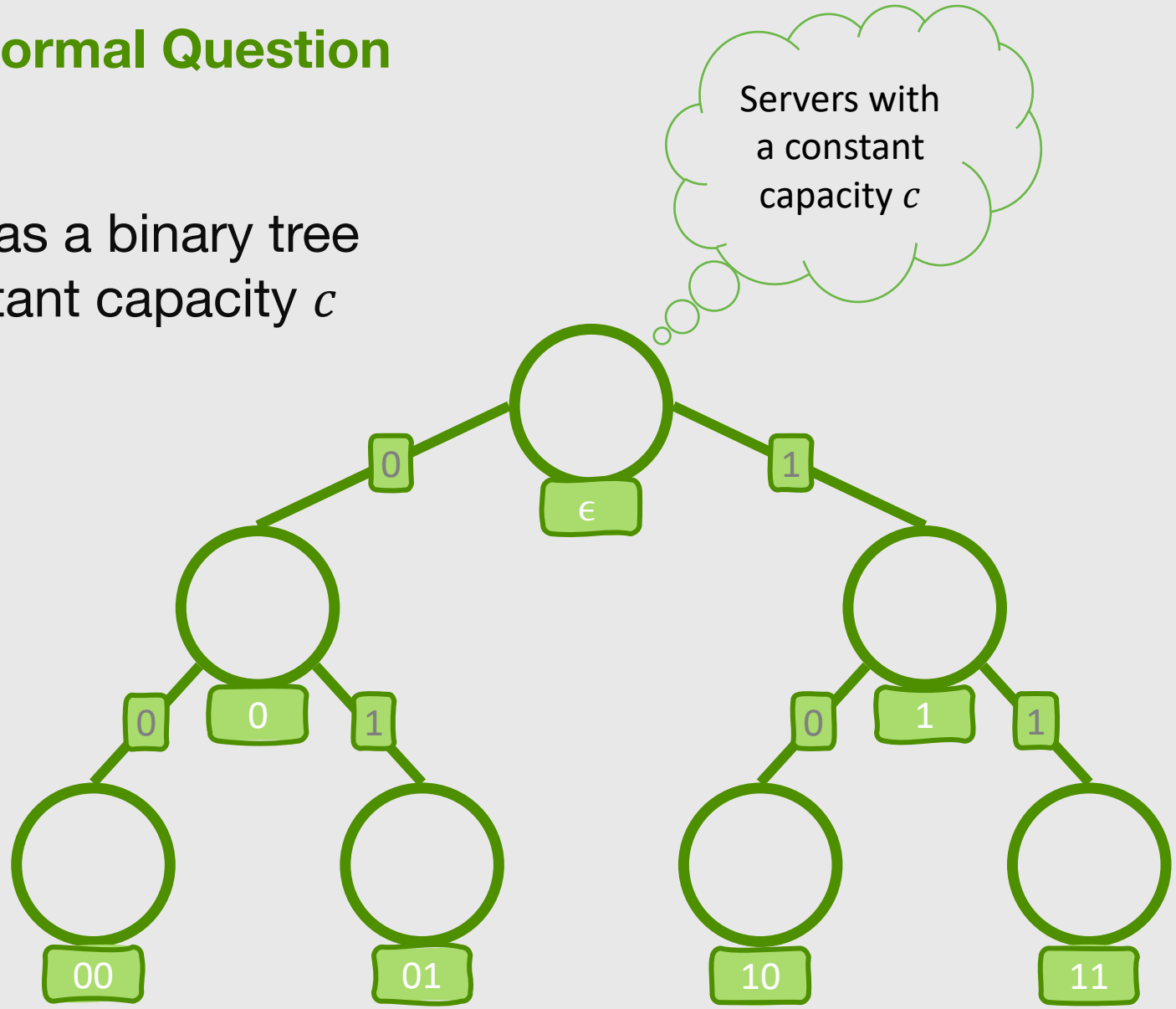
An Abstraction



Formal Question

- Given:
  - A set of servers connected as a binary tree
    - Each server with a constant capacity c



Tree

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- Given:
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    - Revealed over time ($\sigma = \sigma_1, \sigma_2, \ldots$)

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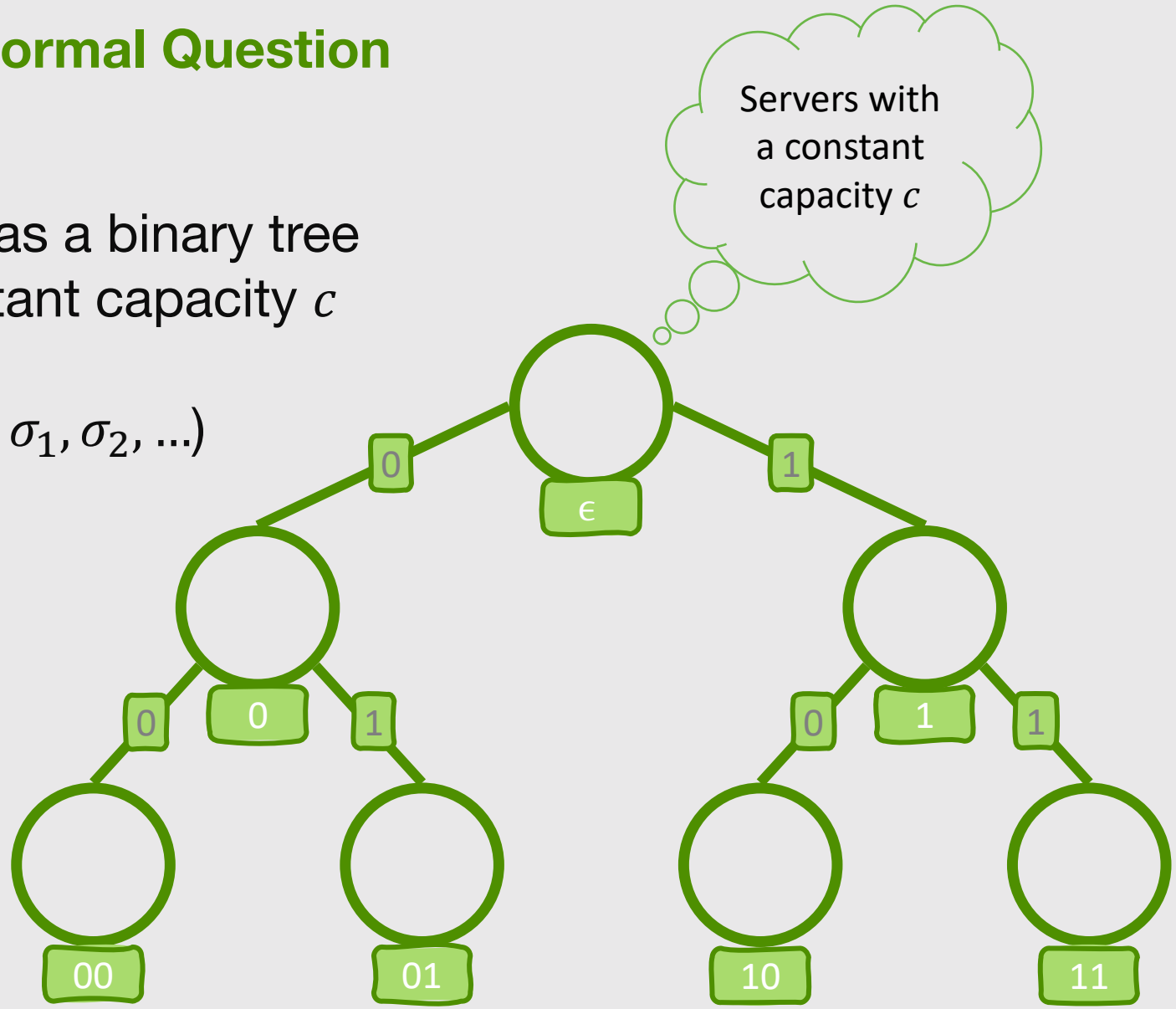

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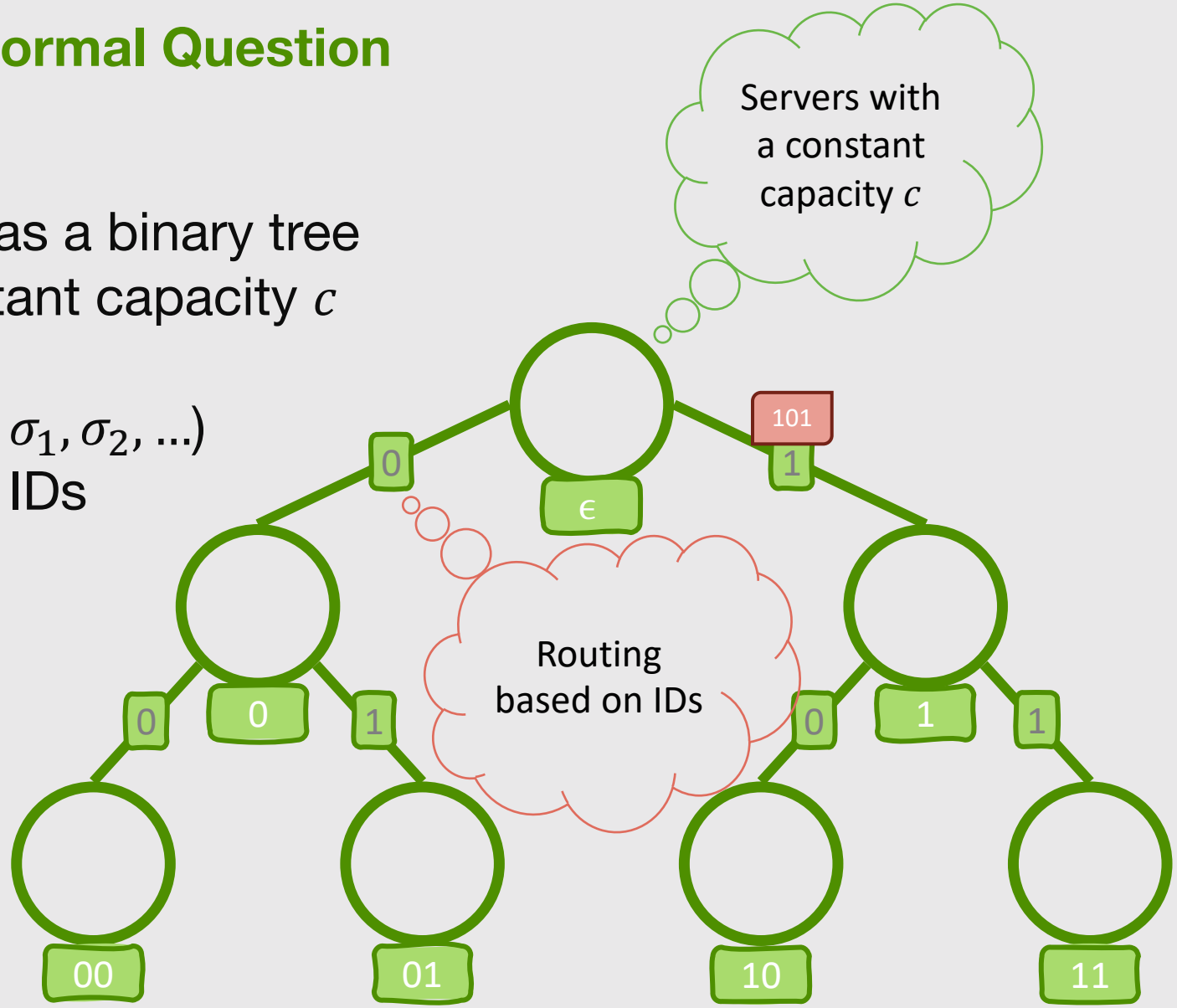

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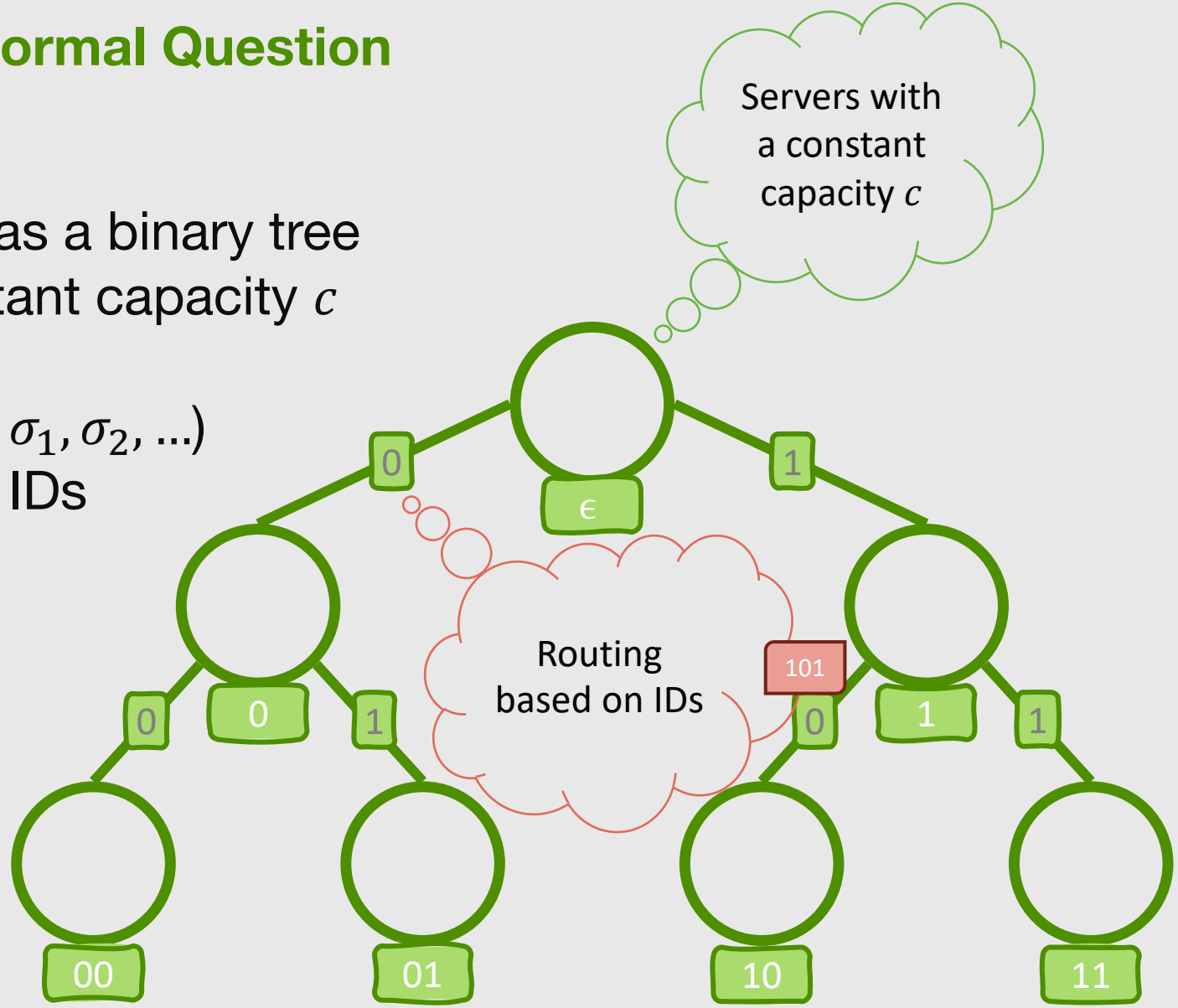

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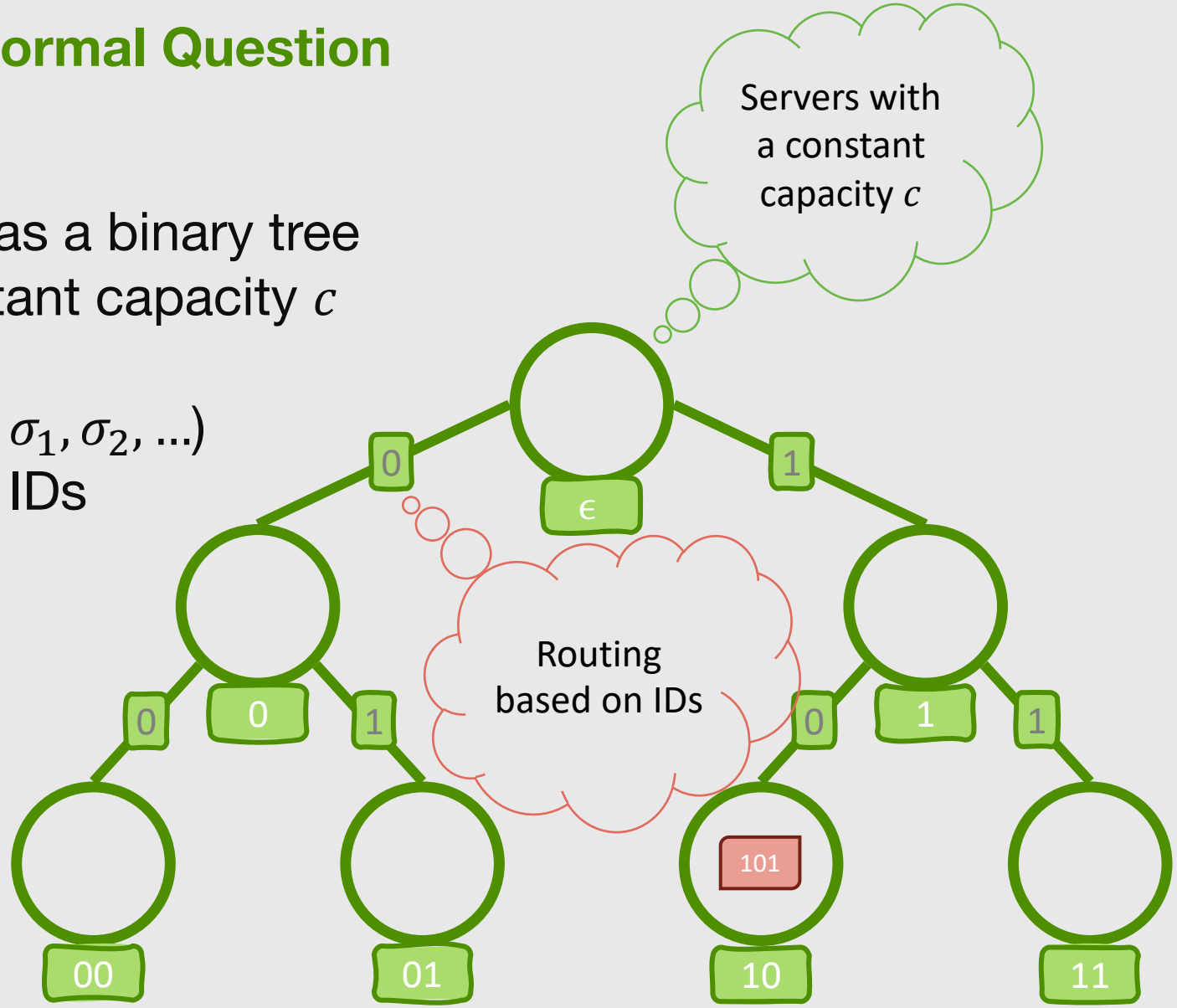

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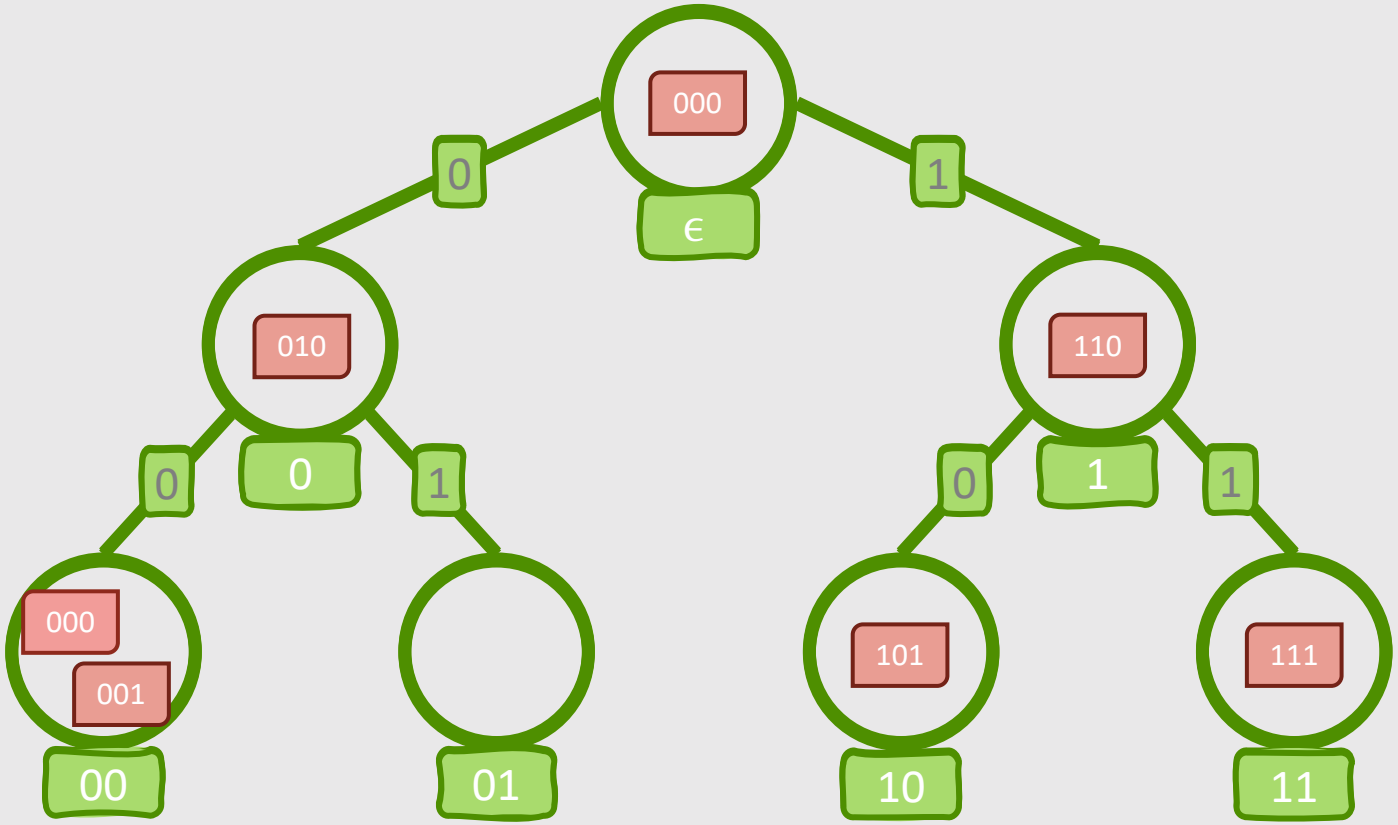

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- Actions (for a prefilled tree*):
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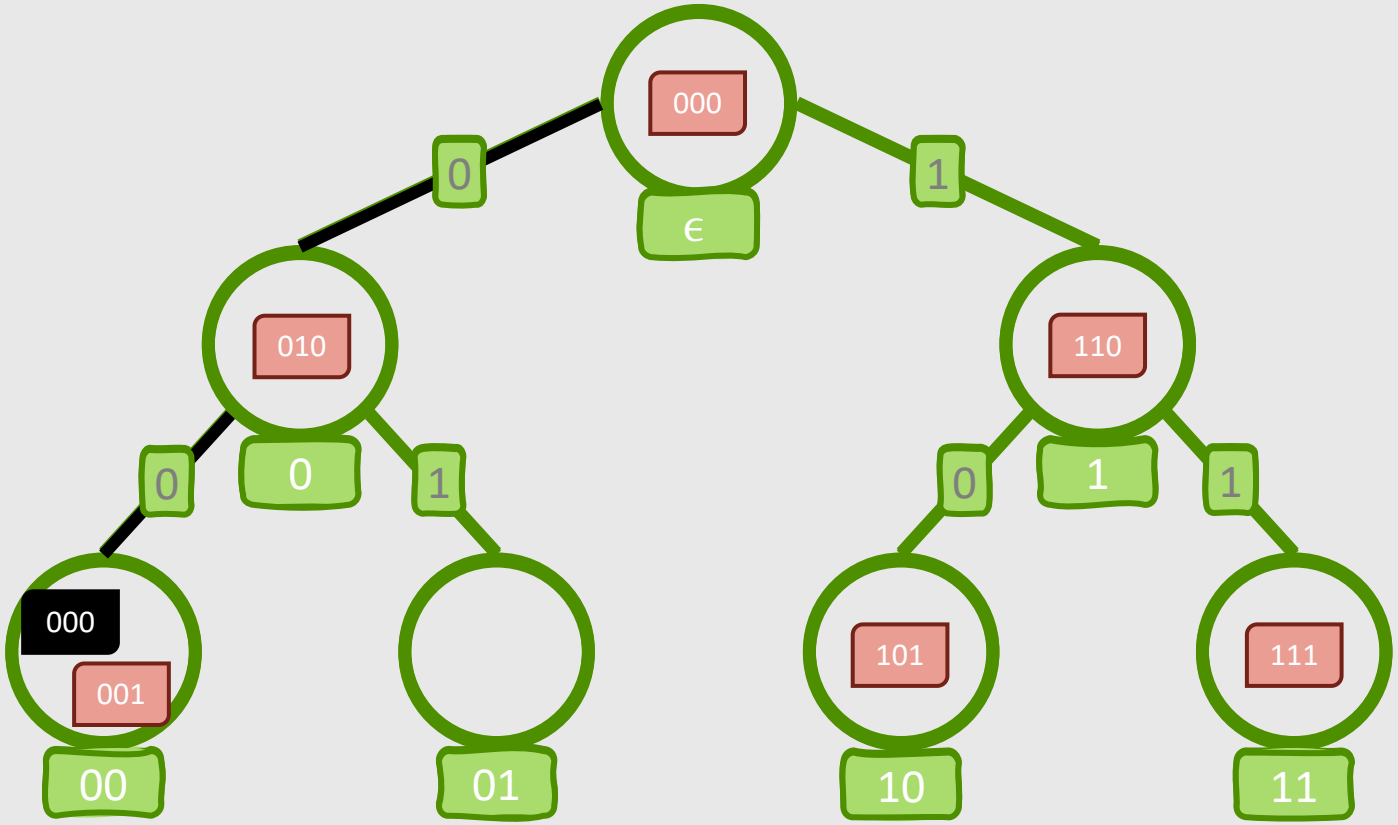

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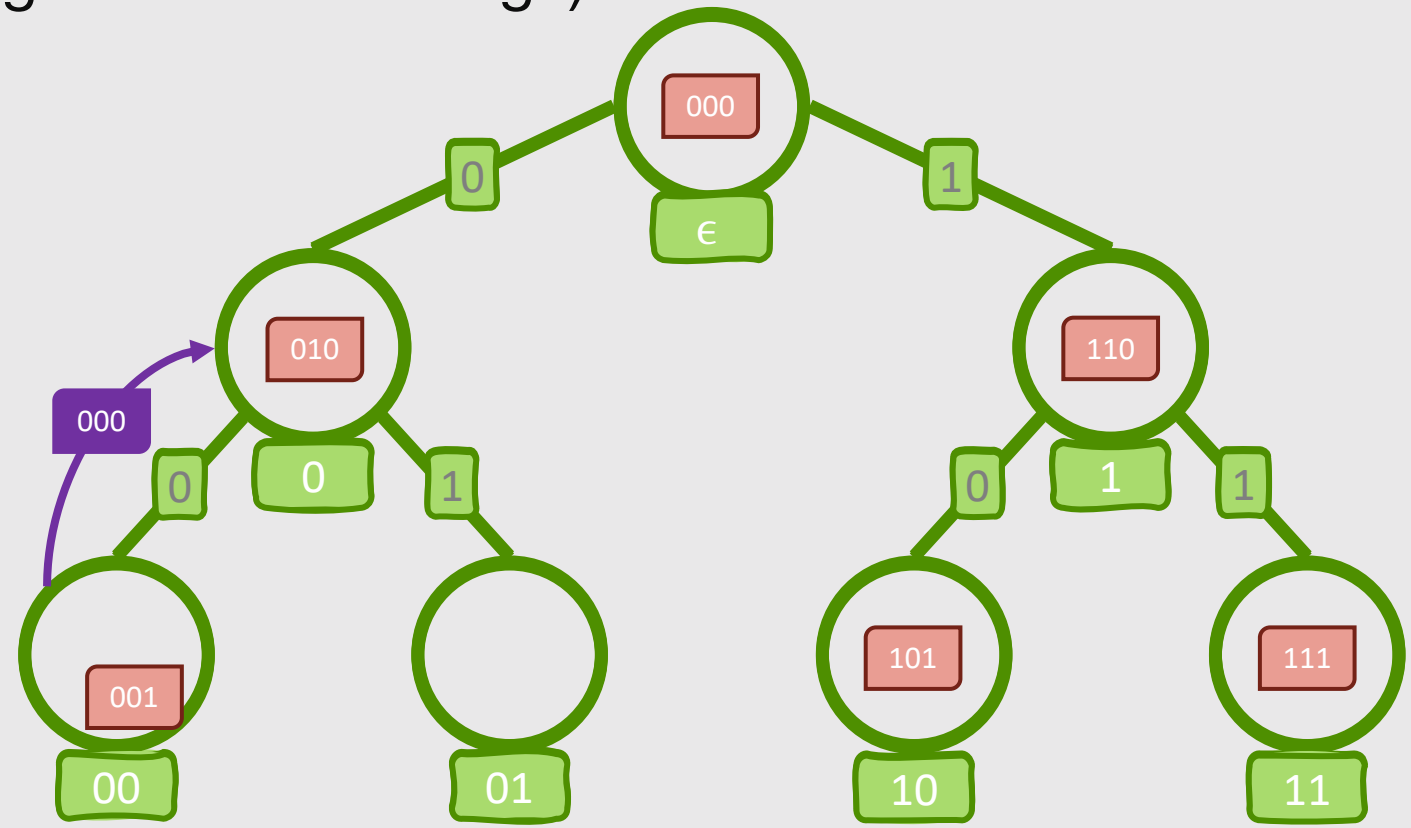

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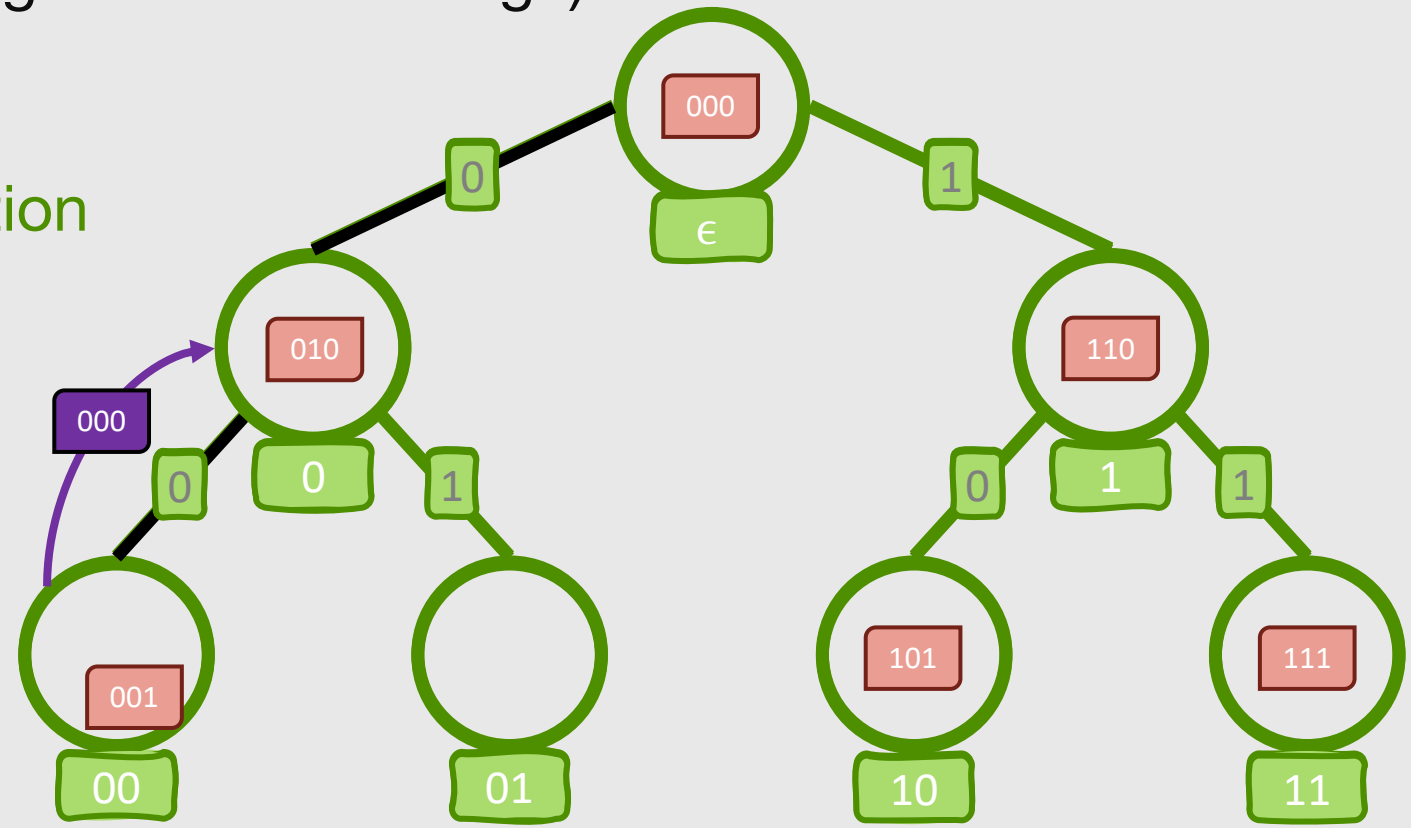

Formal Question

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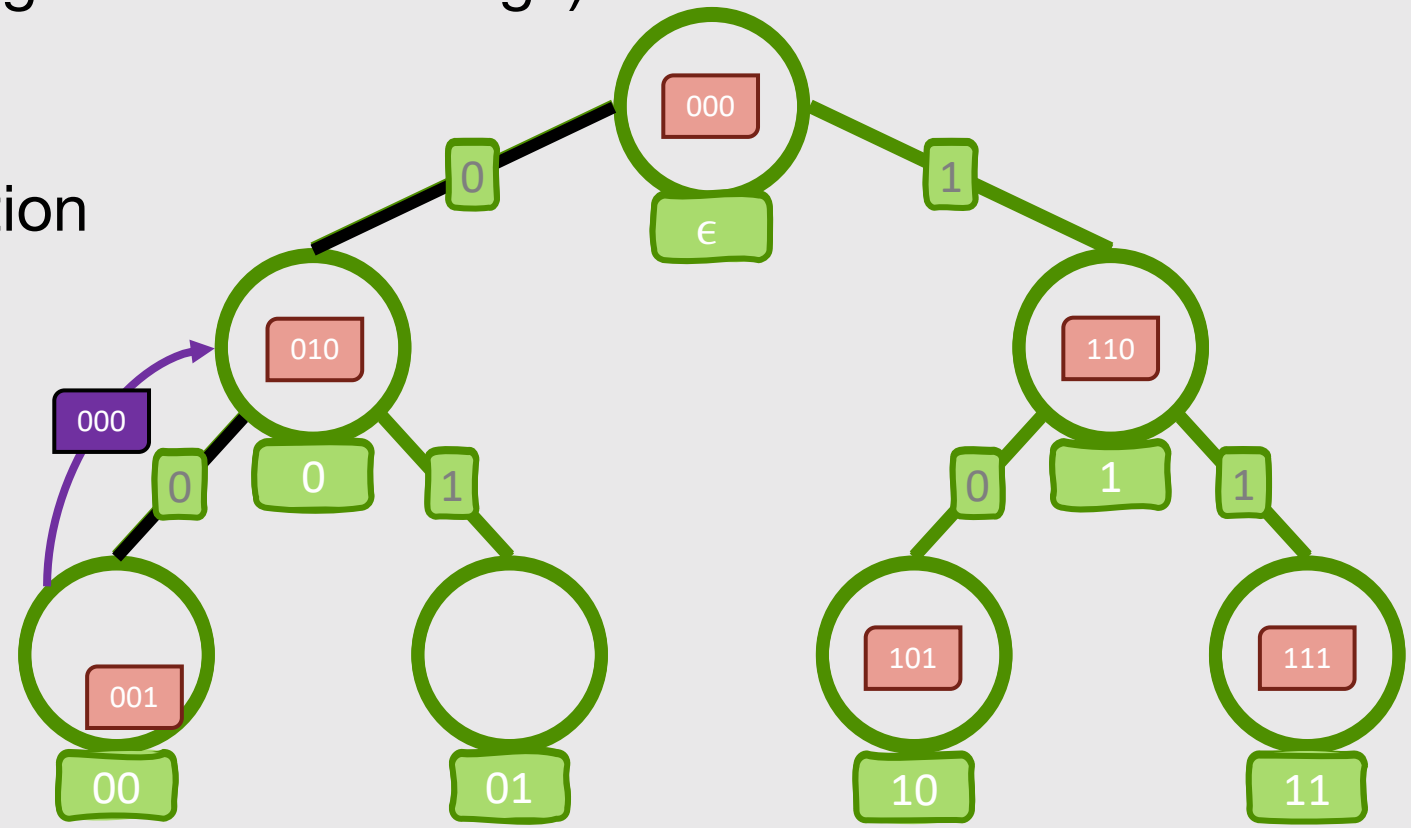
- Dynamically optimal
  - i.e., constant competitive
  - $Cost_{ALG} \leq \alpha \cdot Cost_{OPT}$



Previous work

Data structure	Operation	Dynamically Optimal	Local Routing?
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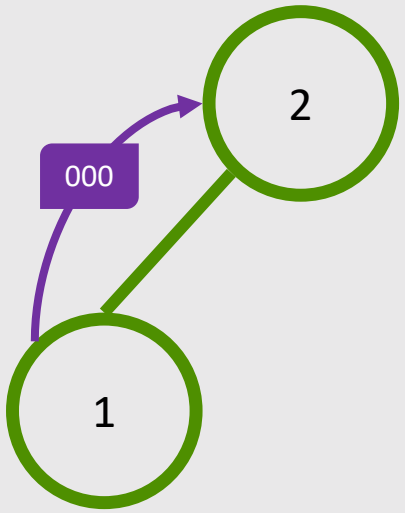

Moving Items

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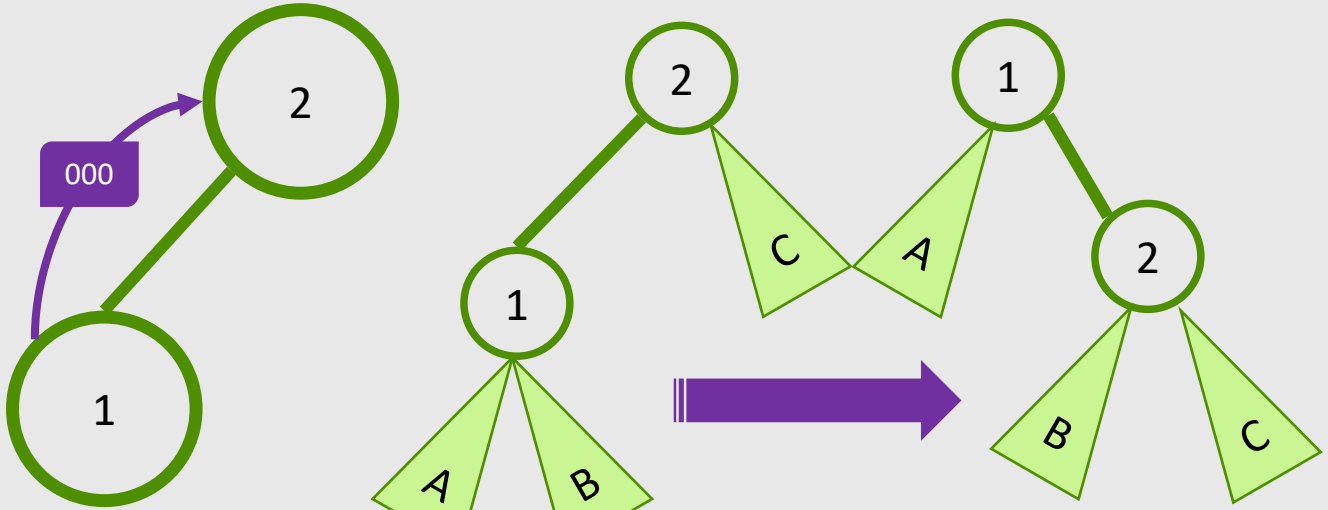

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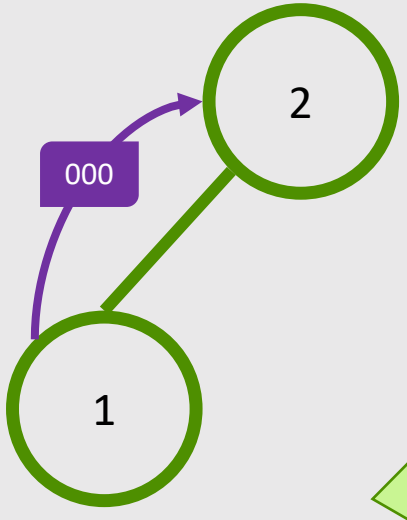
Adaptive Huffman [Vitter J.ACM'87, FGK J.ACM'85]	Subtree Swap	✅	❌



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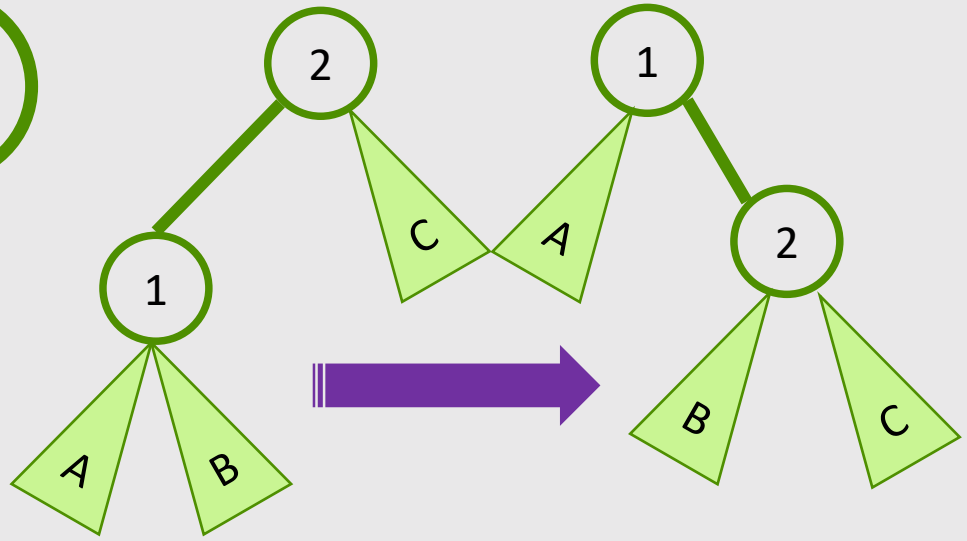
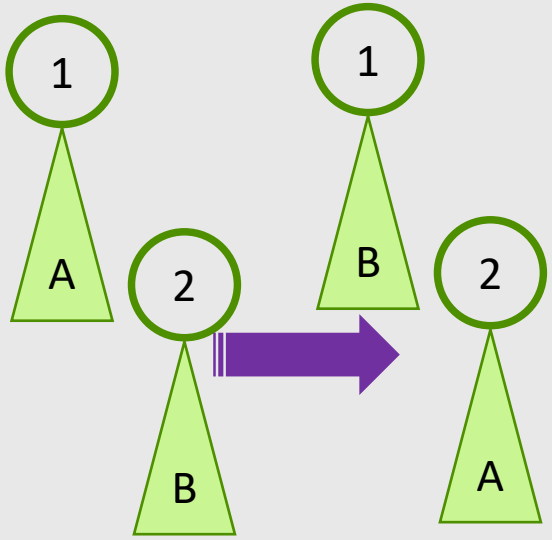


Rotation



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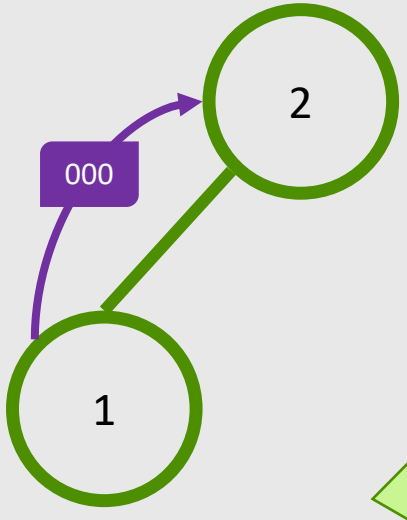
Adaptive Huffman [Vitter J.ACM'87, FGK J.ACM'85]	Subtree Swap	✅	❌
Push-down-Tree [Avin et al. LATIN'20, TON'22]	Item Swap	✅	❌



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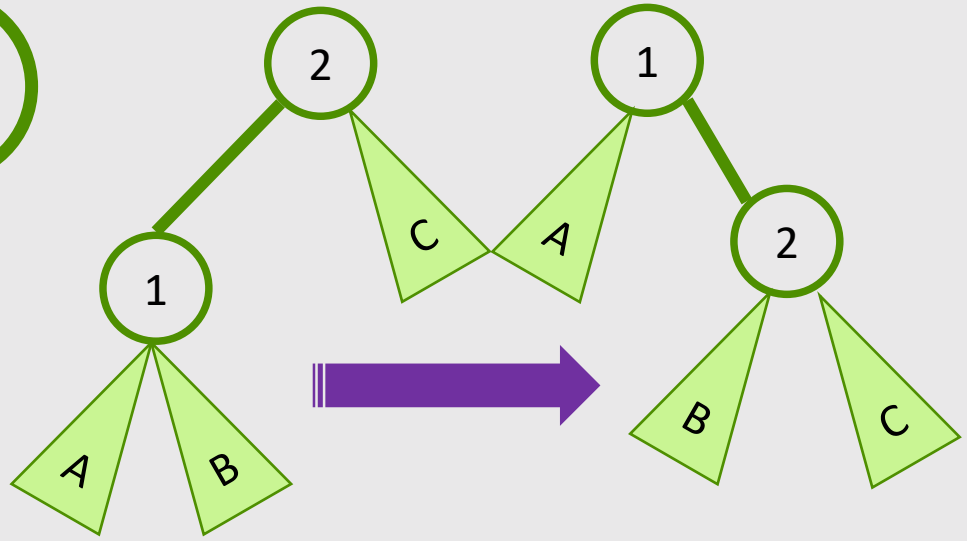

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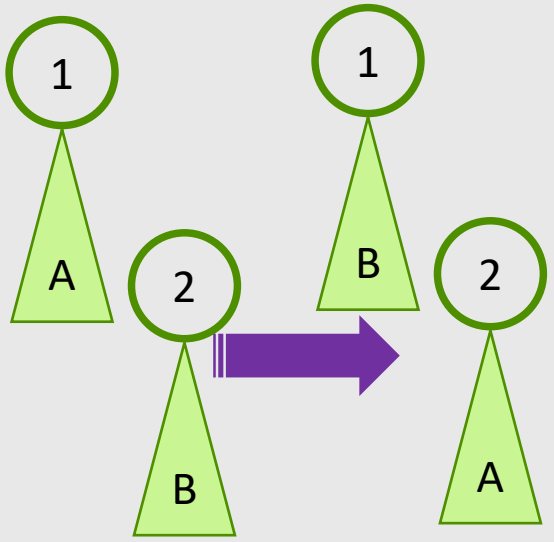

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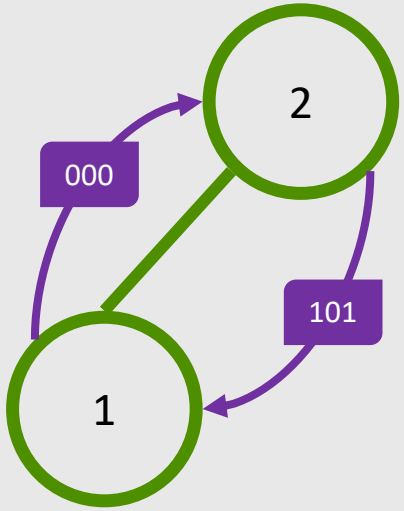

Item Swap

SeedTree

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2) What?

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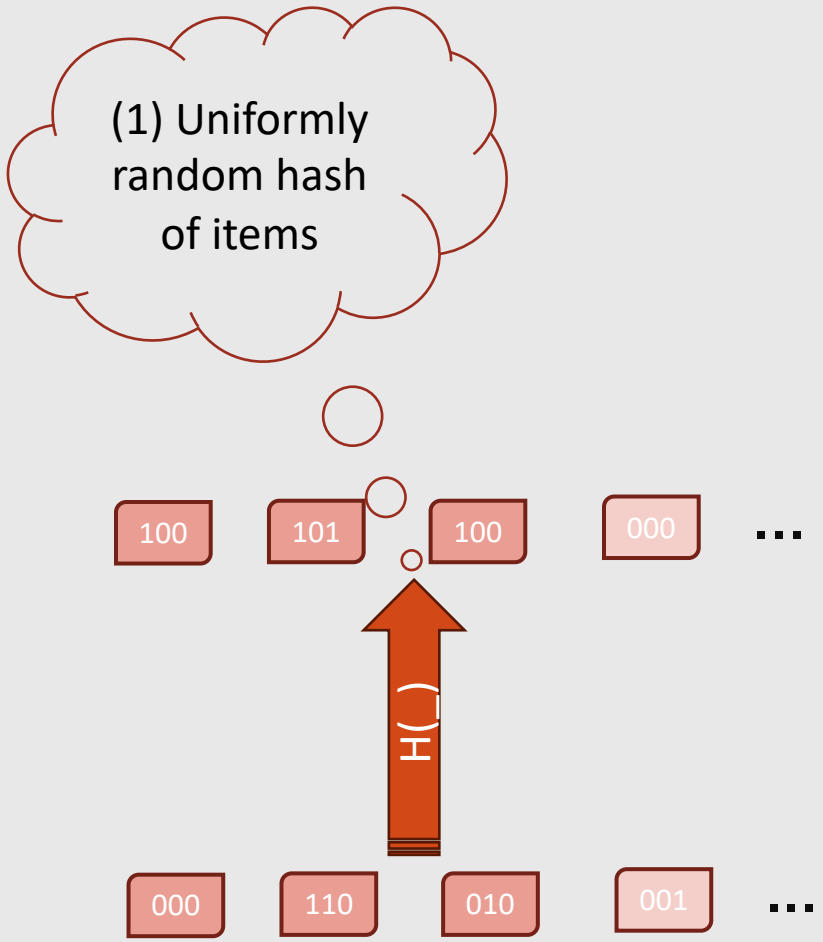
Tools & Techniques



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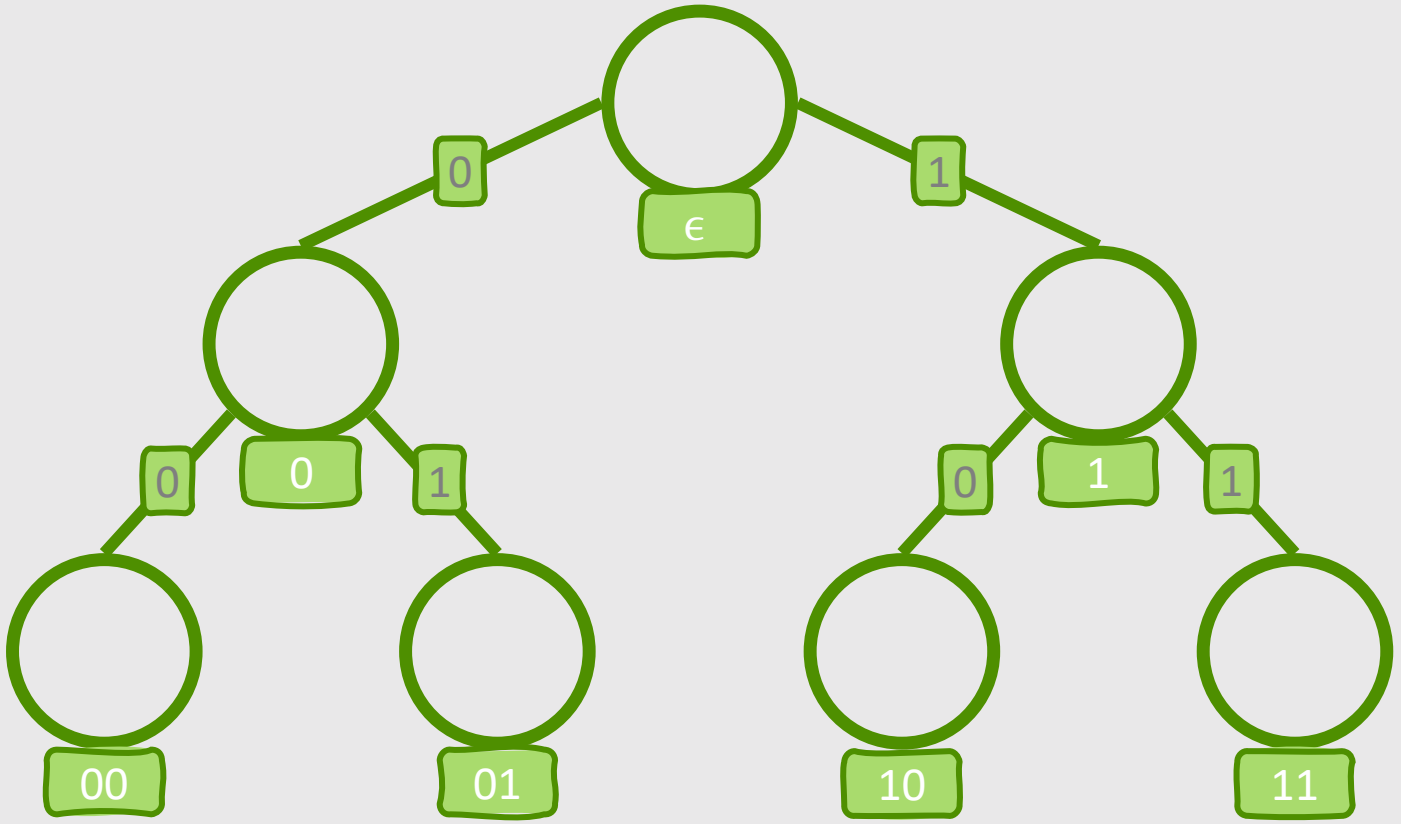
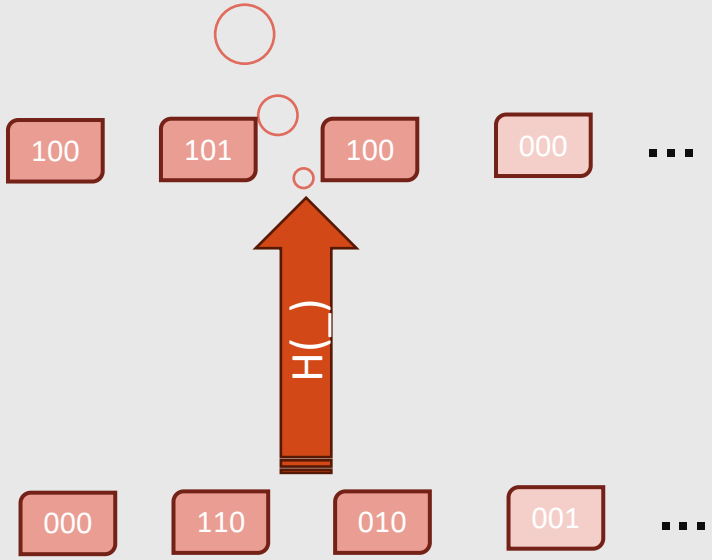


Tree

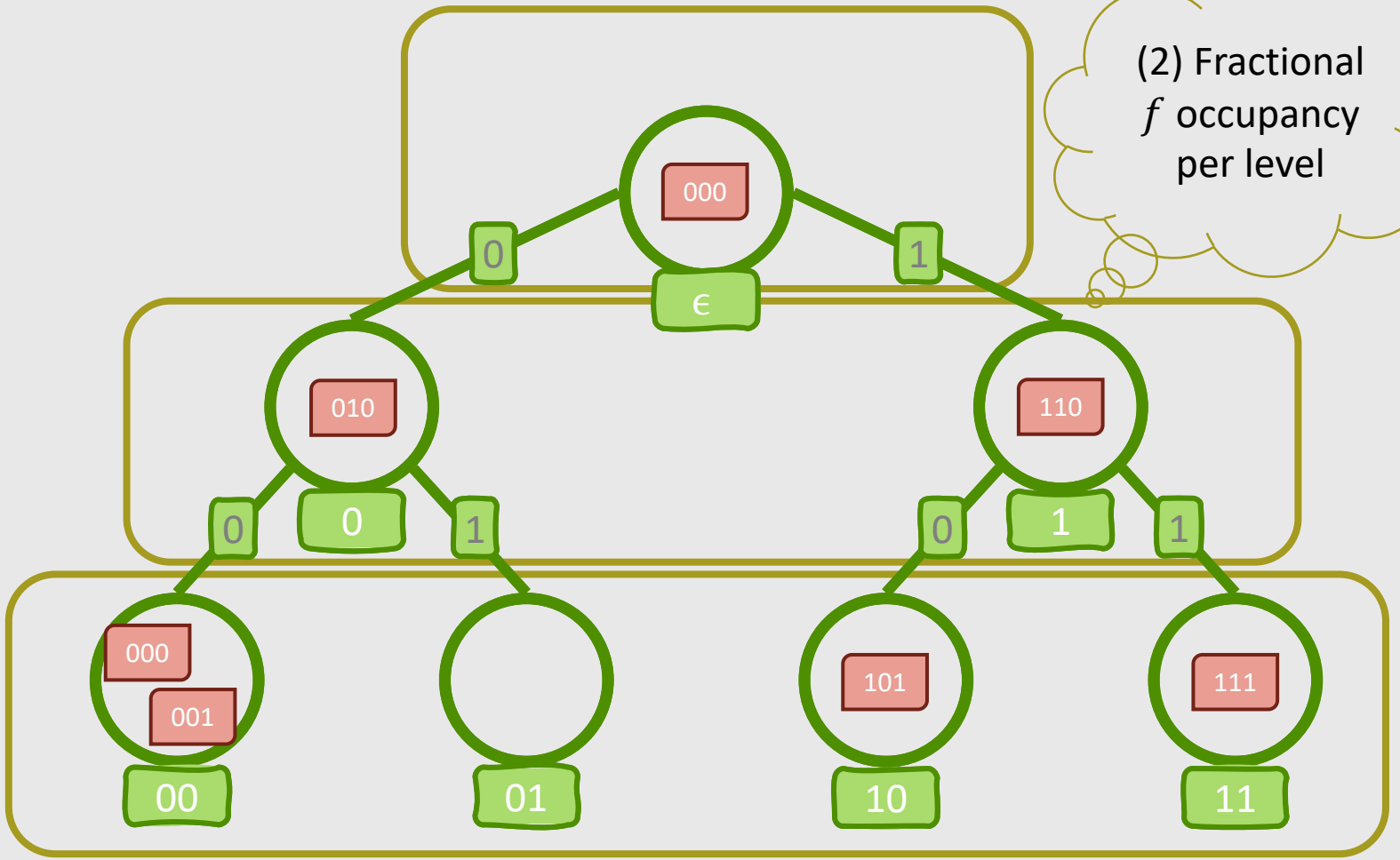
Tools & Techniques

(1) Uniformly random hash of items



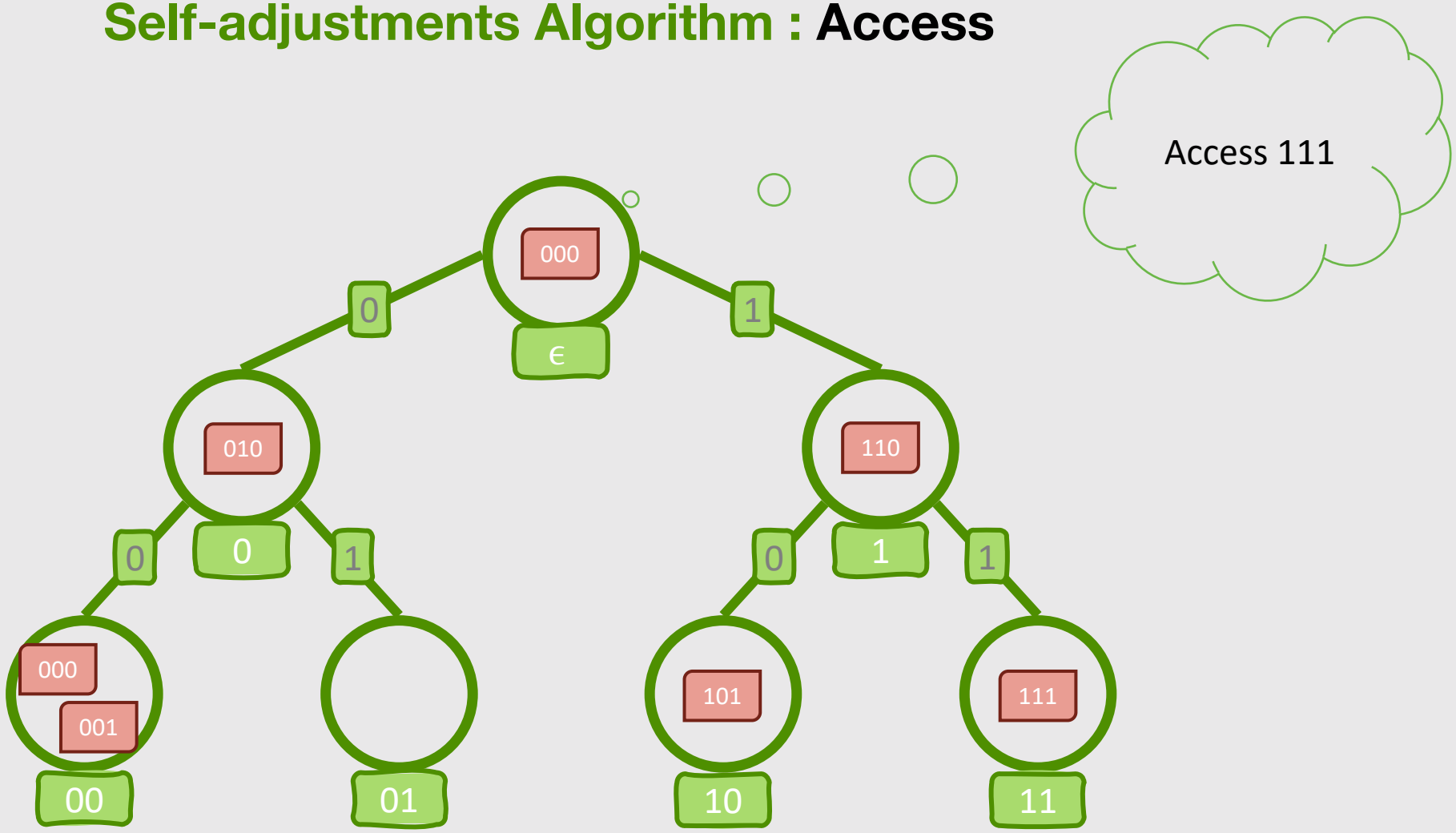
Items

(2) Fractional f occupancy per level



Tree

Self-adjustments Algorithm : Access



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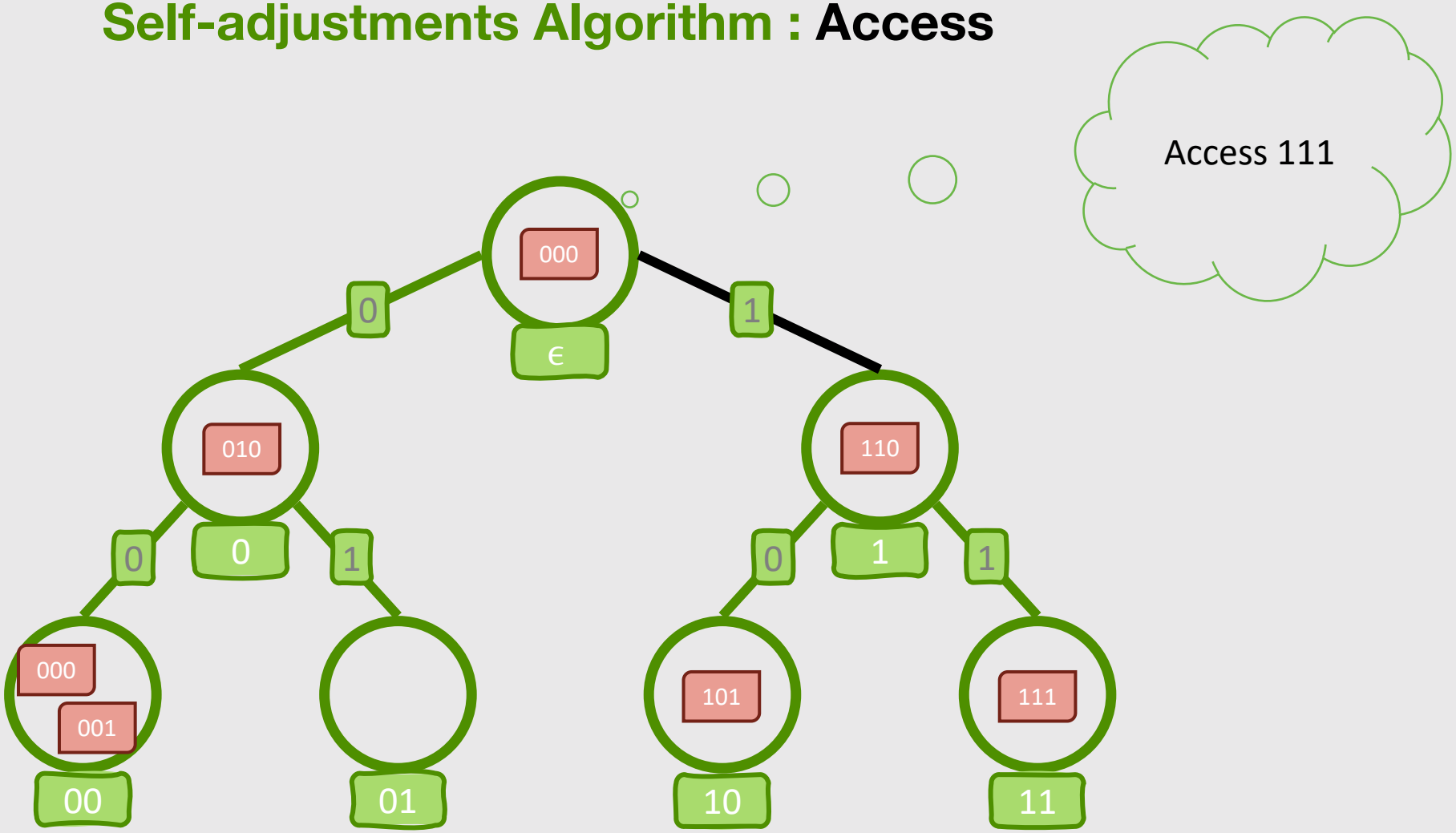

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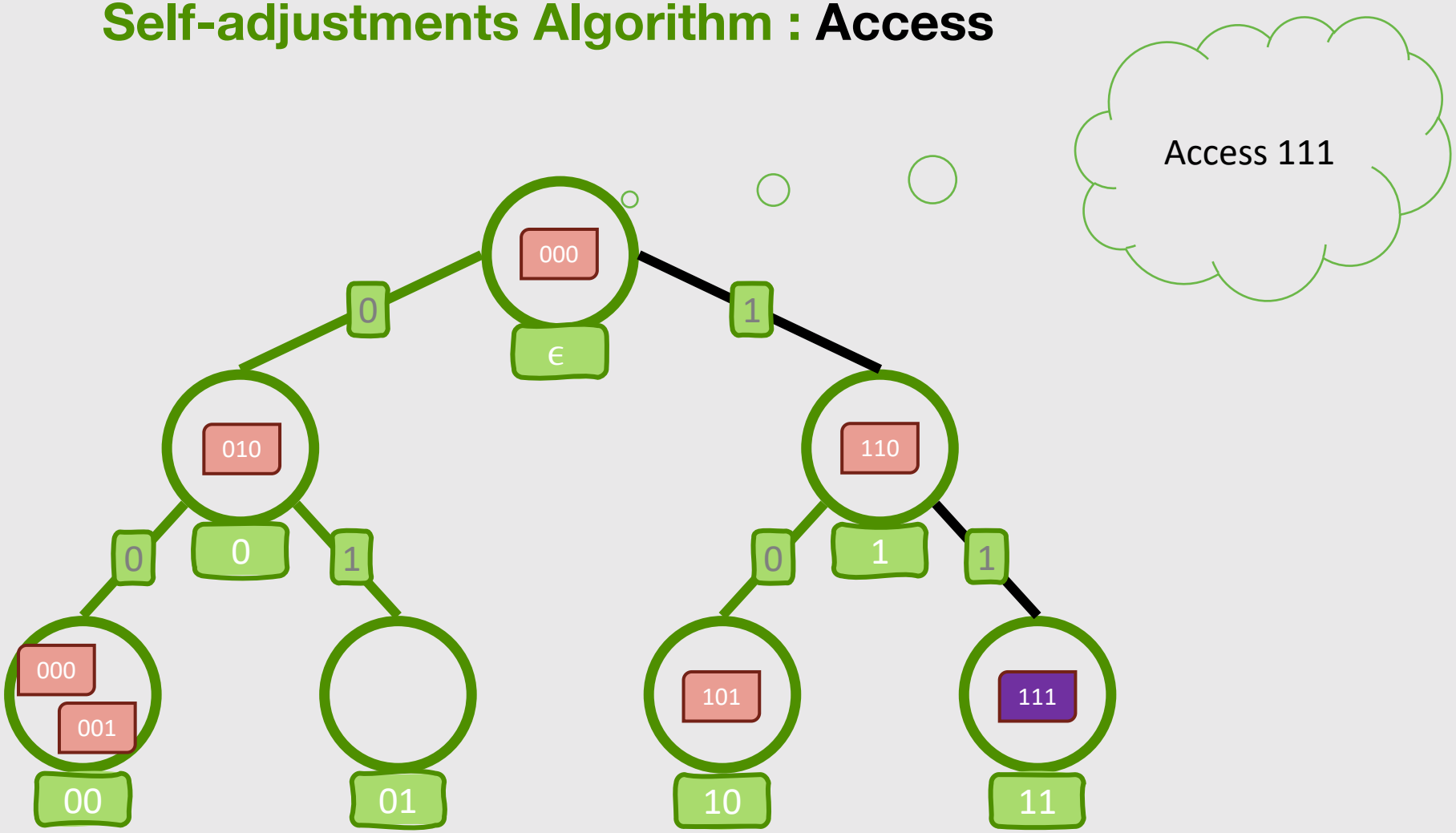
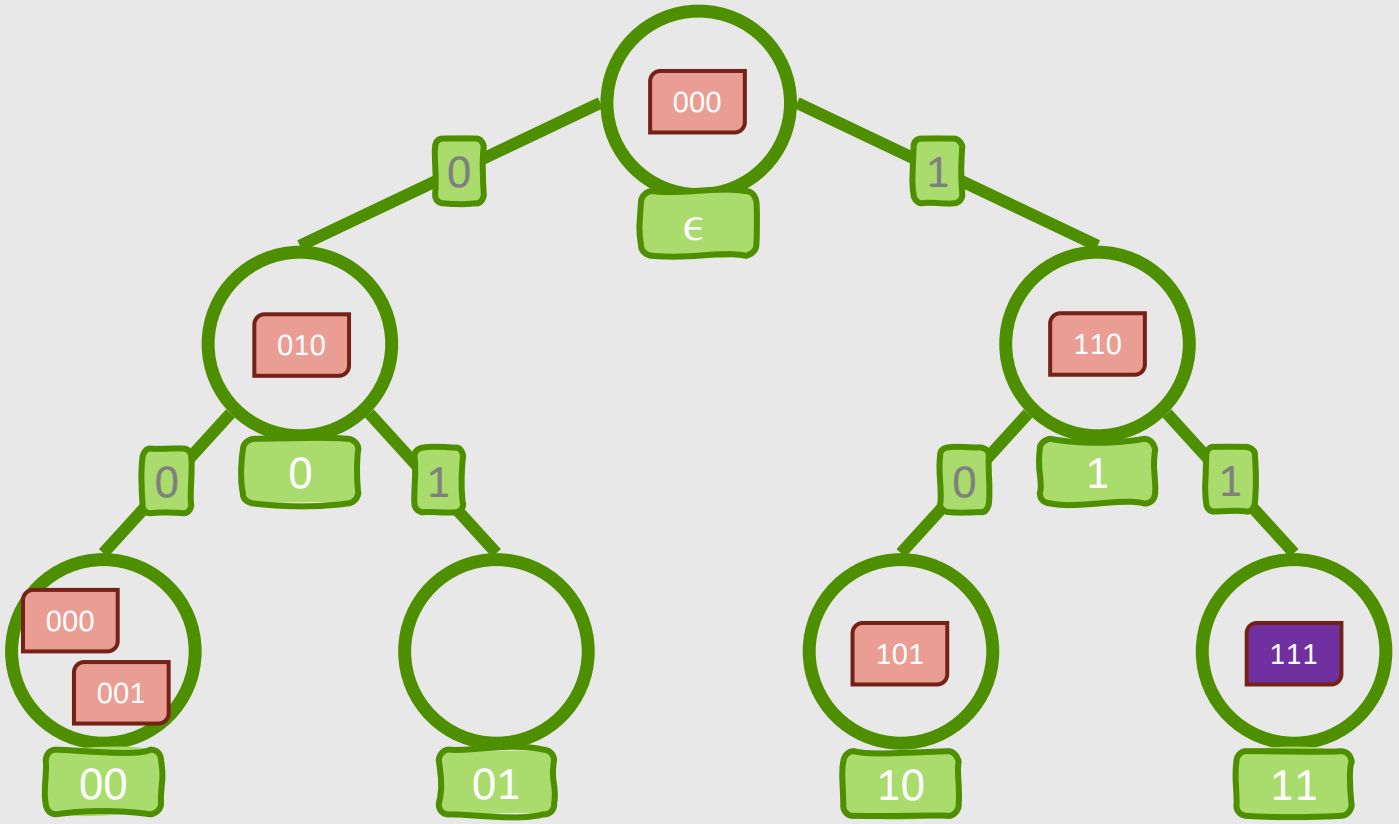


Self-adjustments Algorithm : Pull-up



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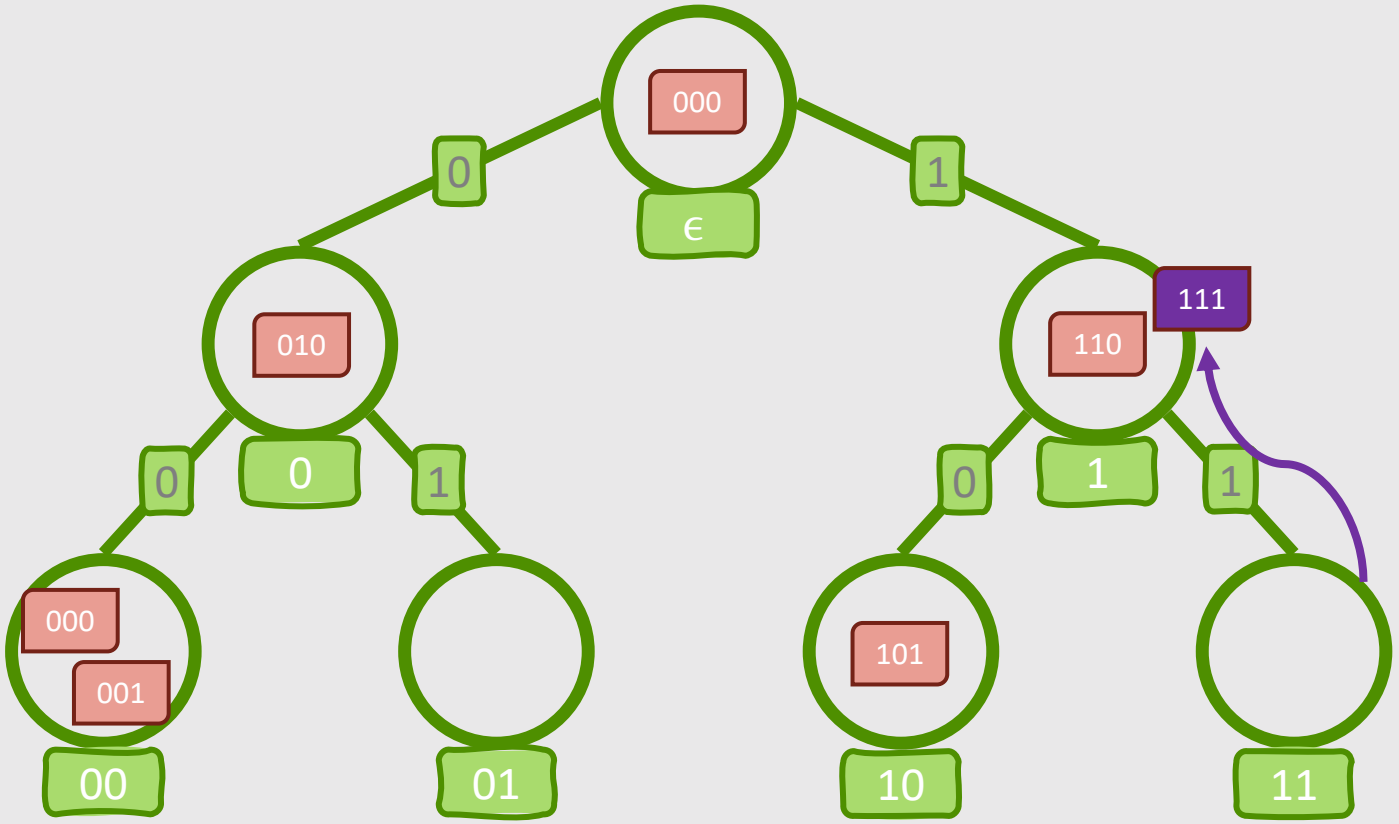

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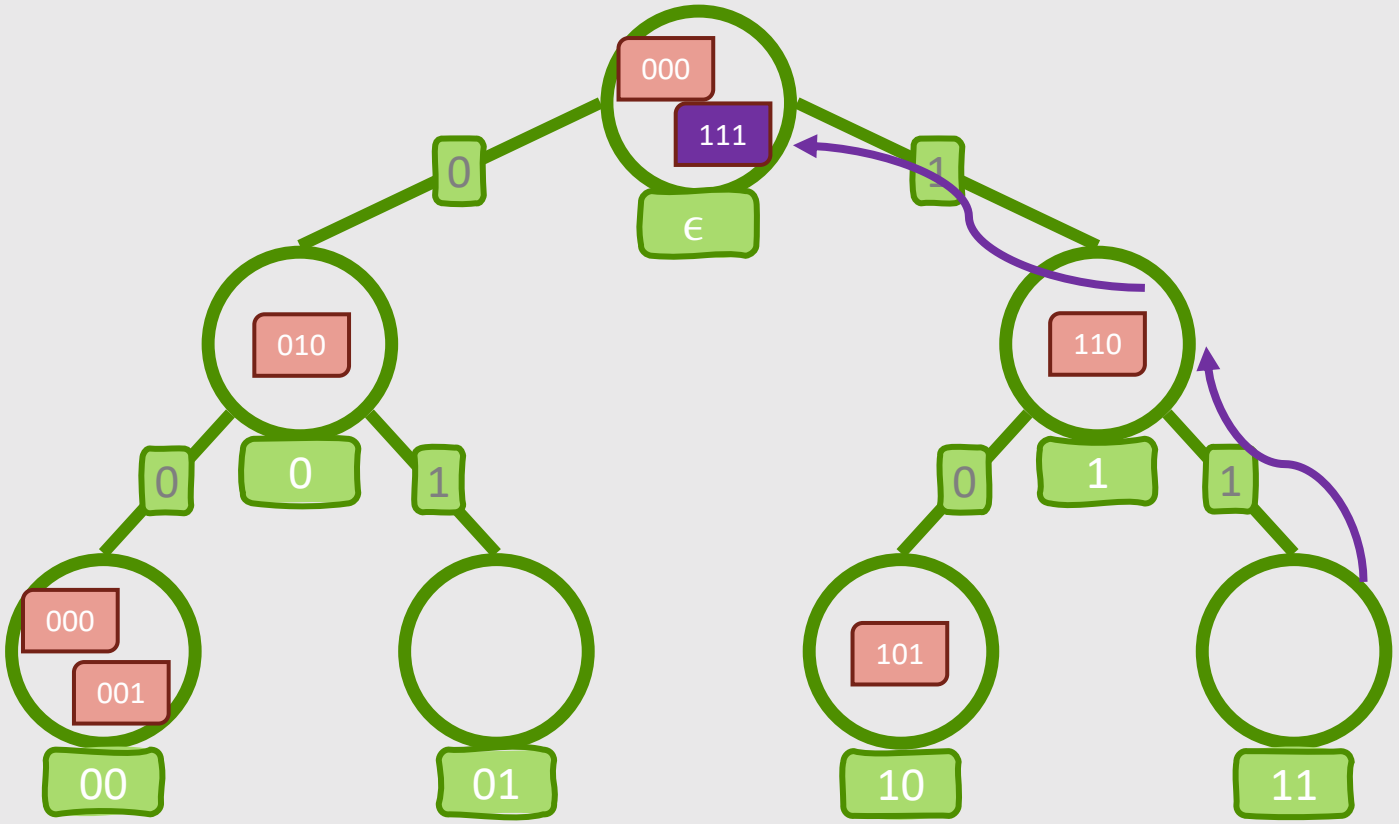
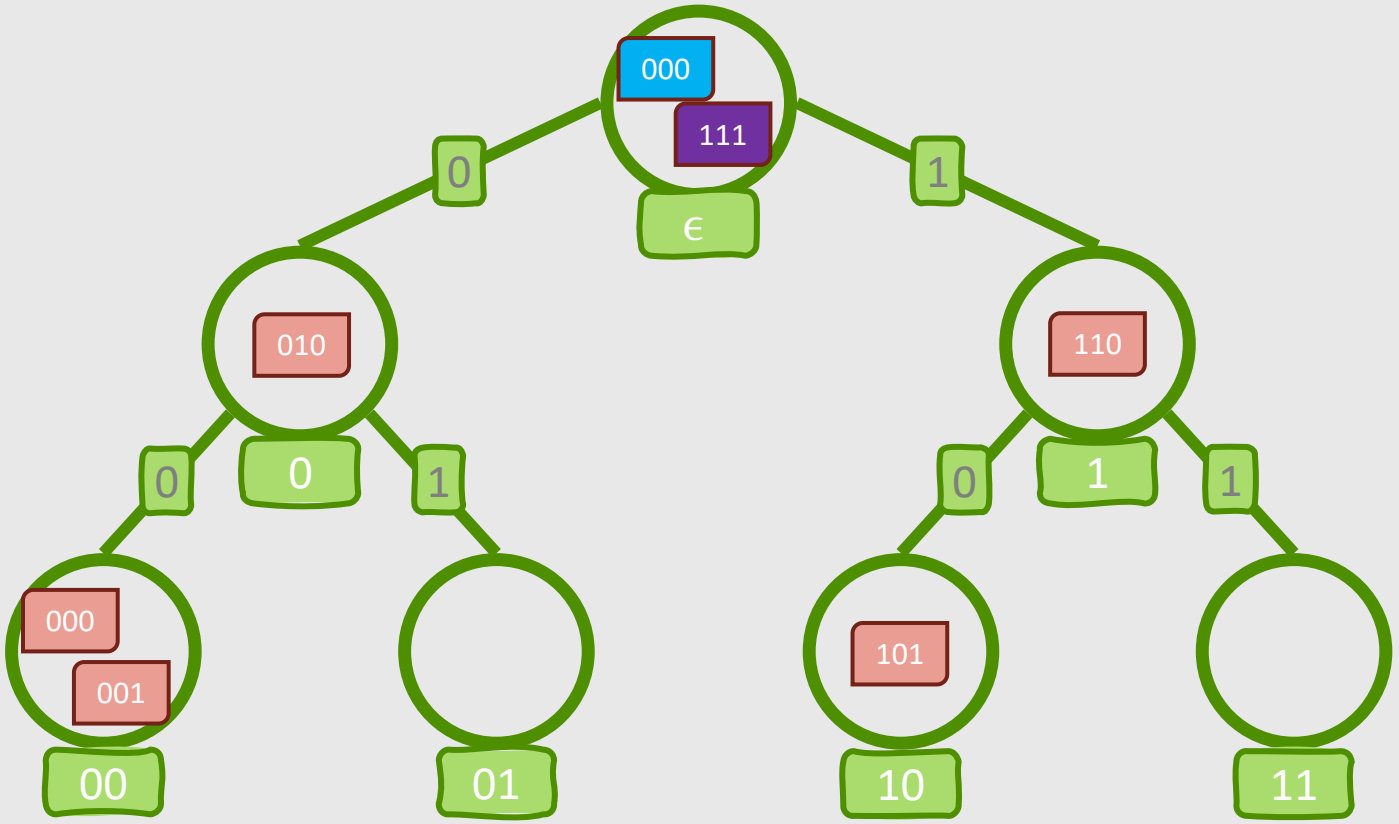


Self-adjustments Algorithm: Push-down



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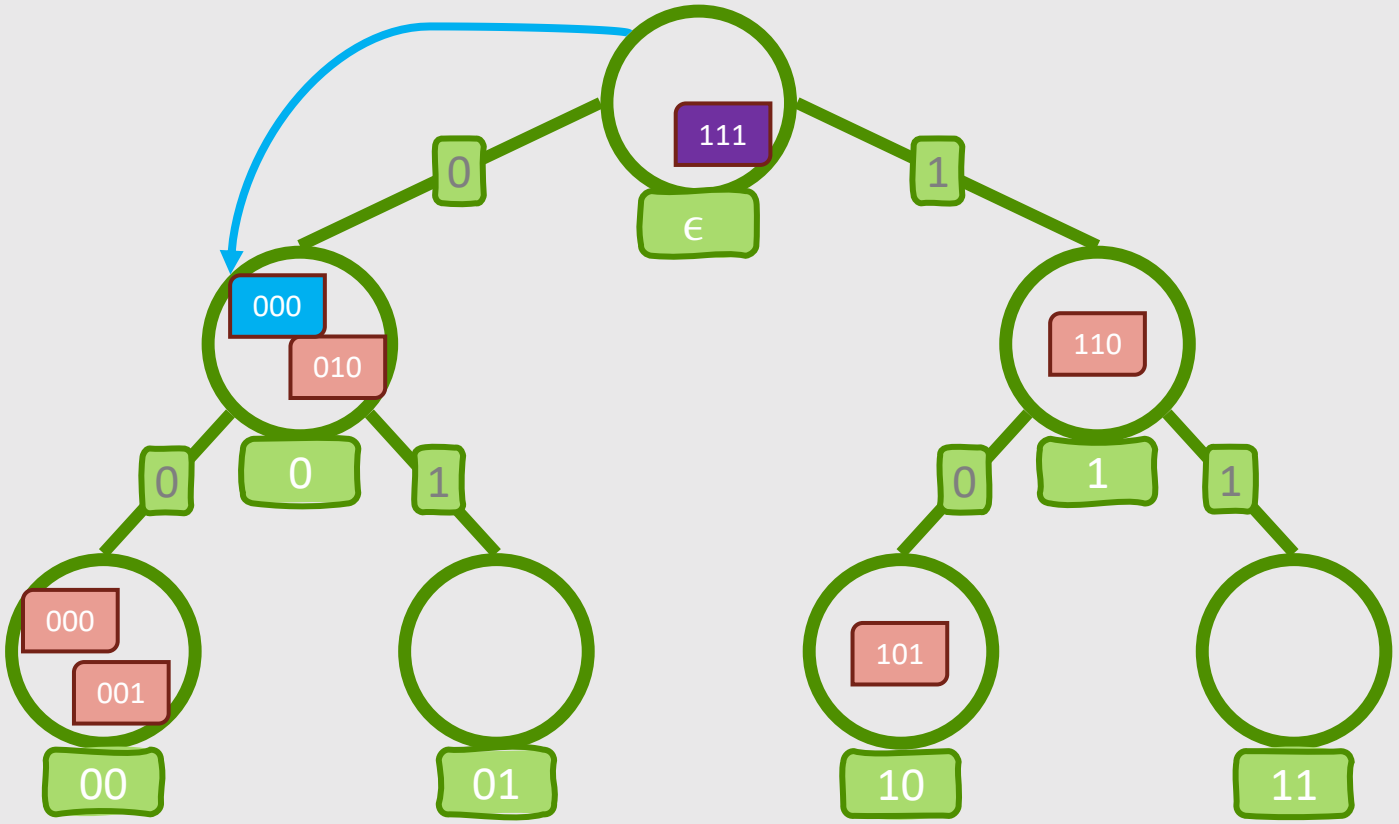

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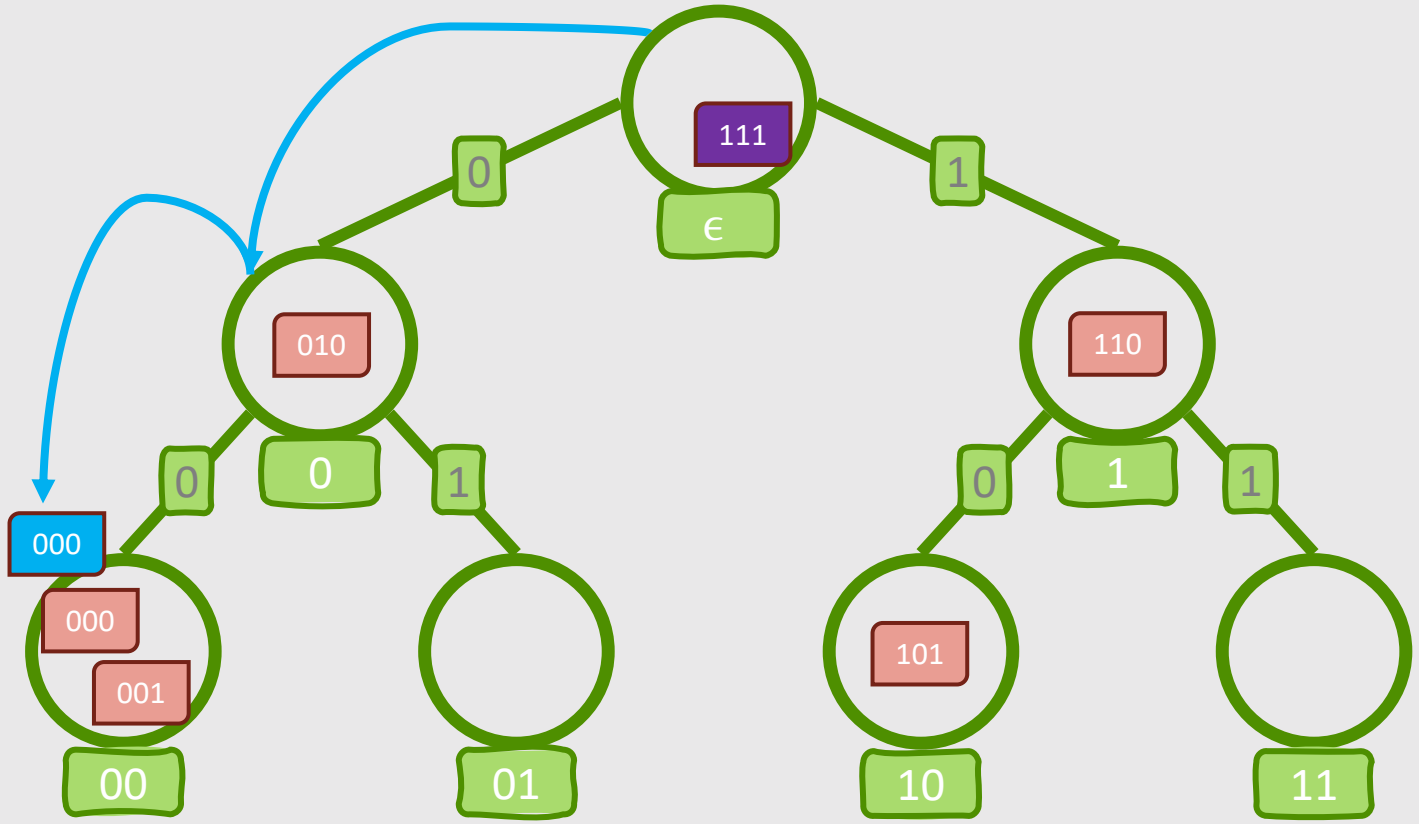

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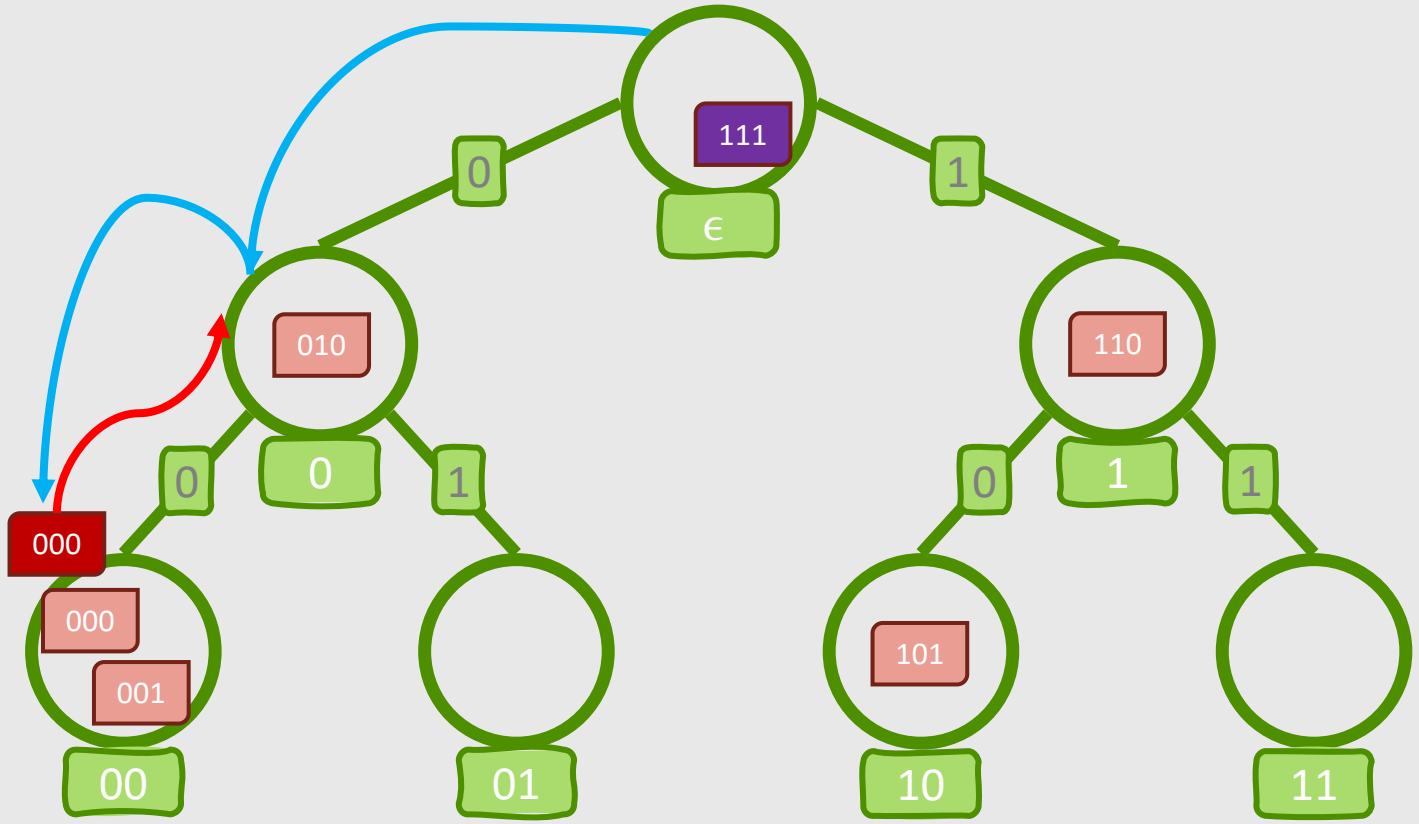

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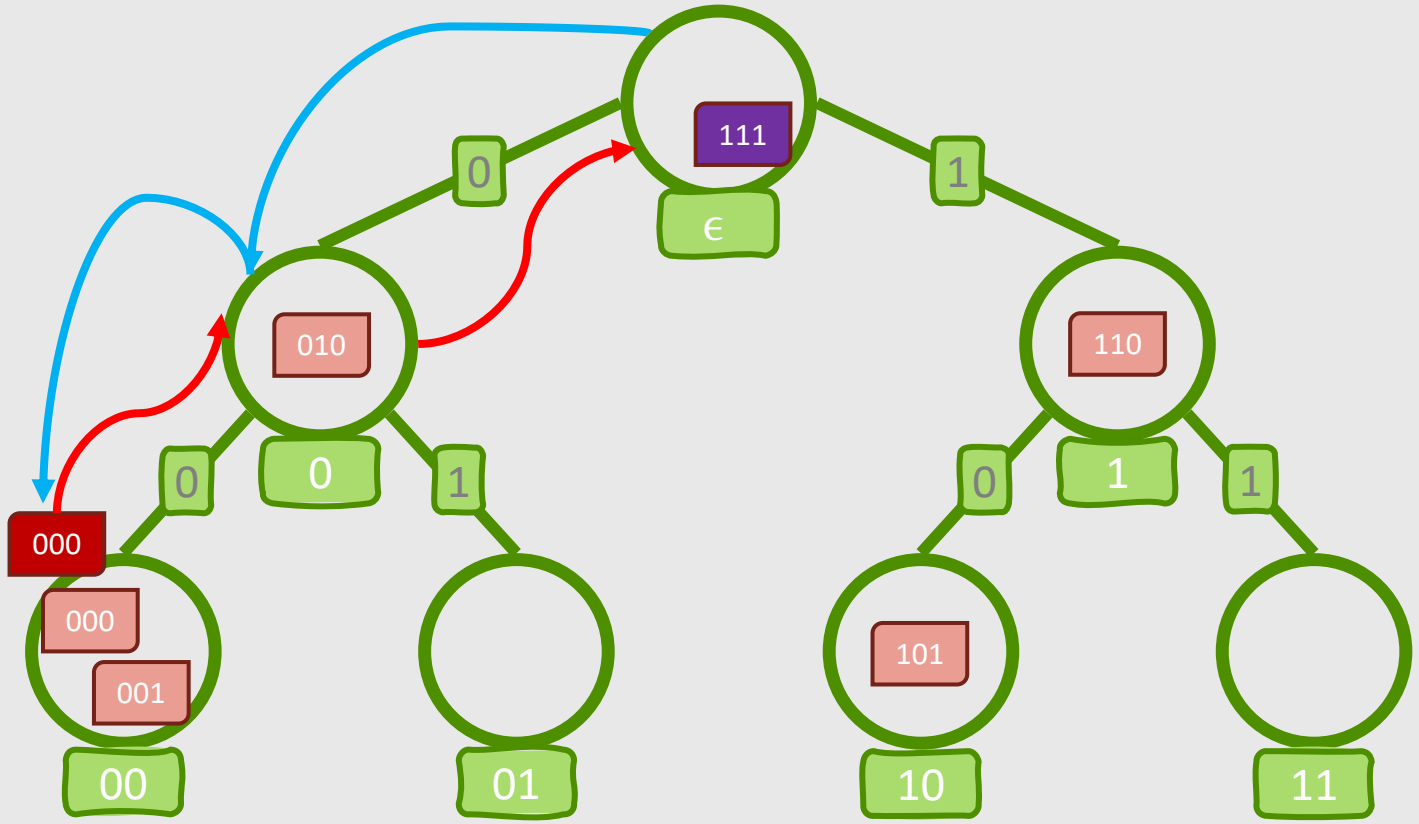

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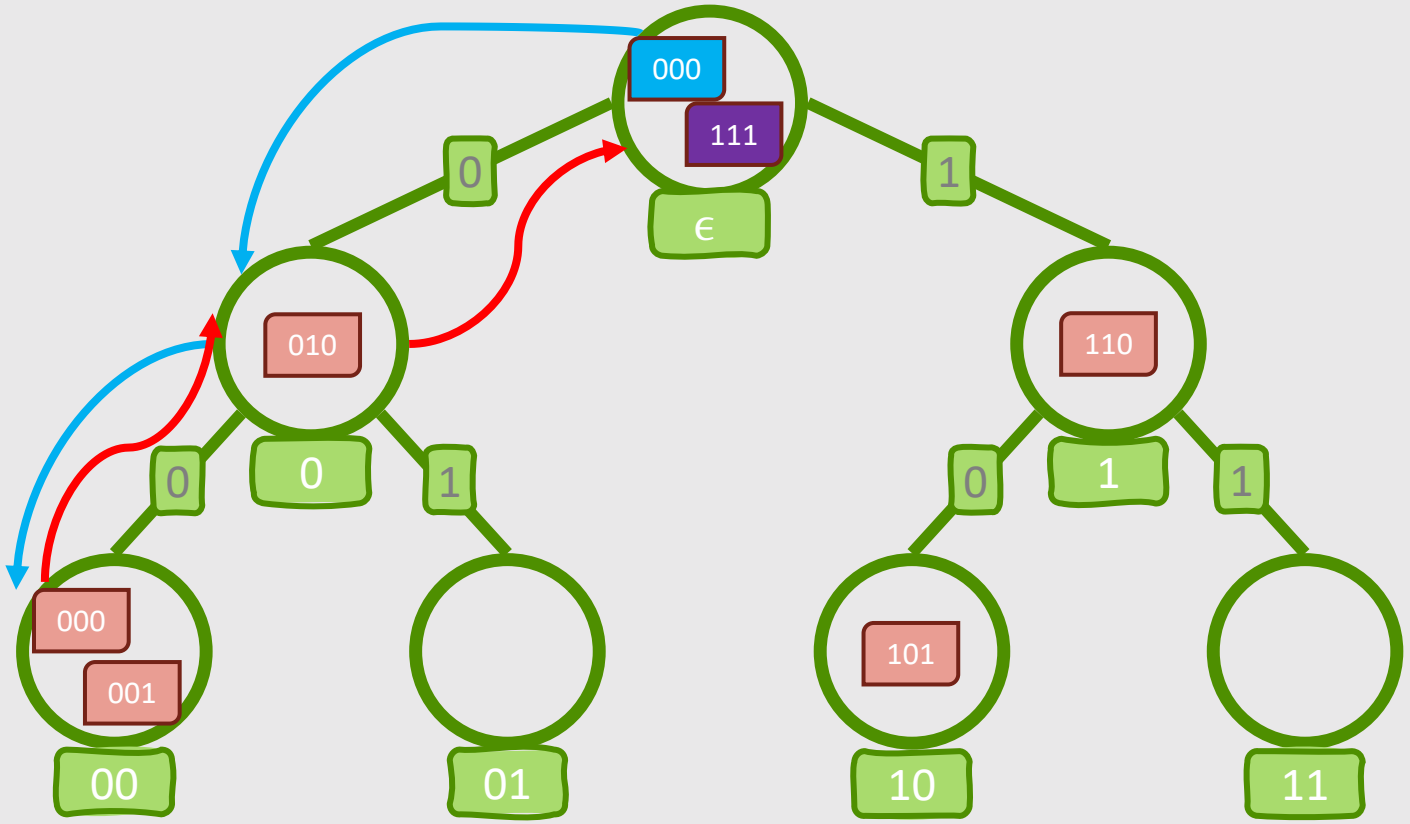

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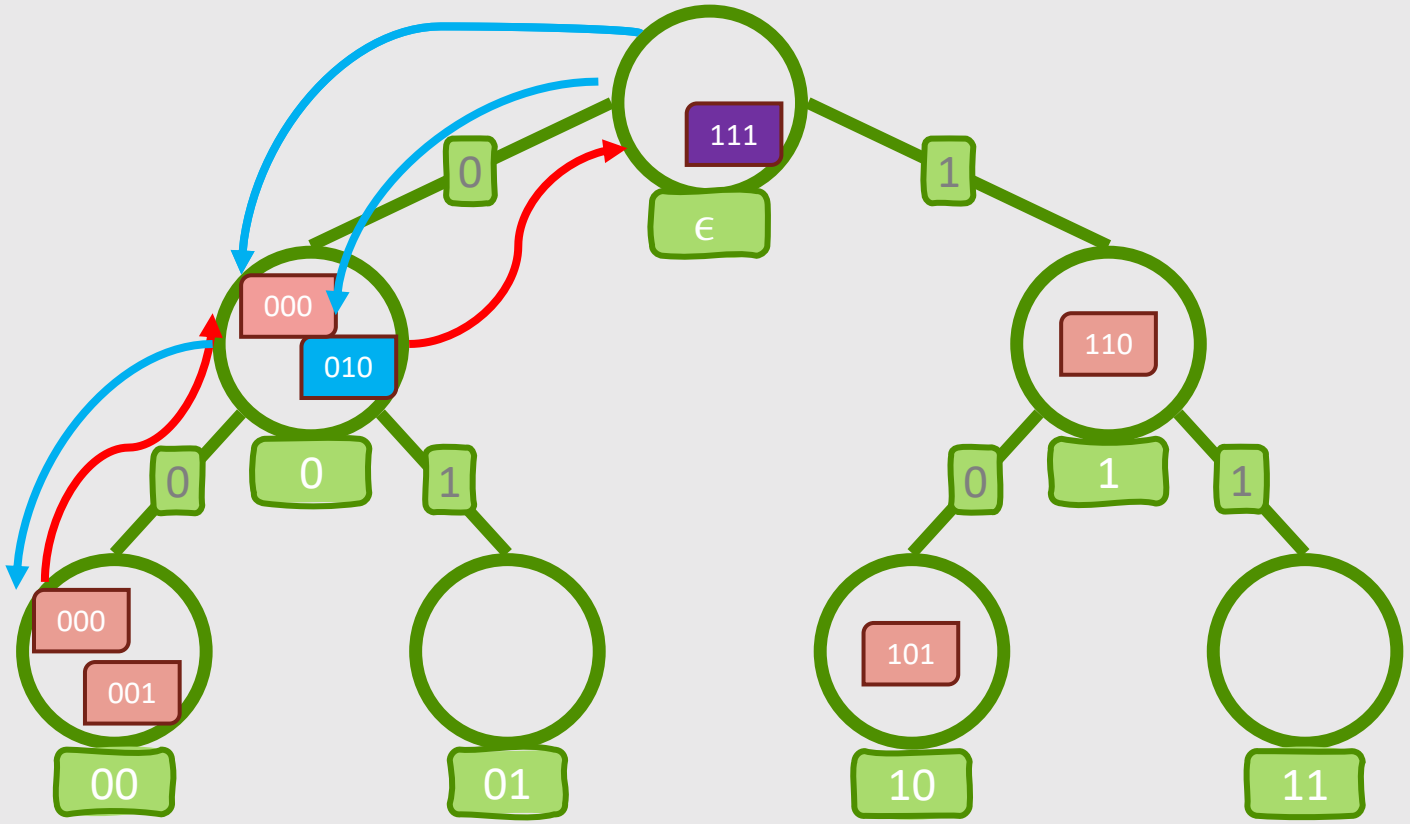

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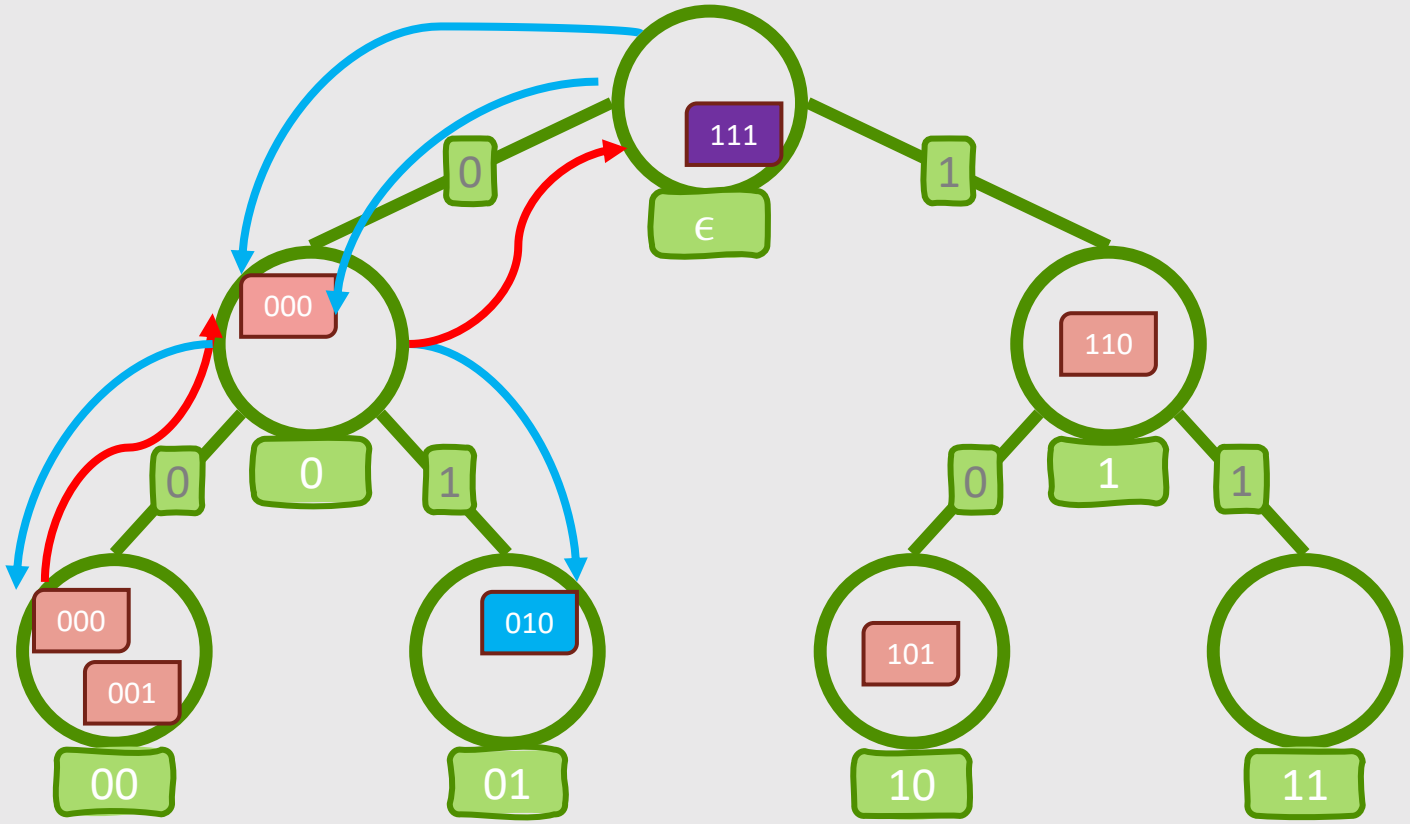

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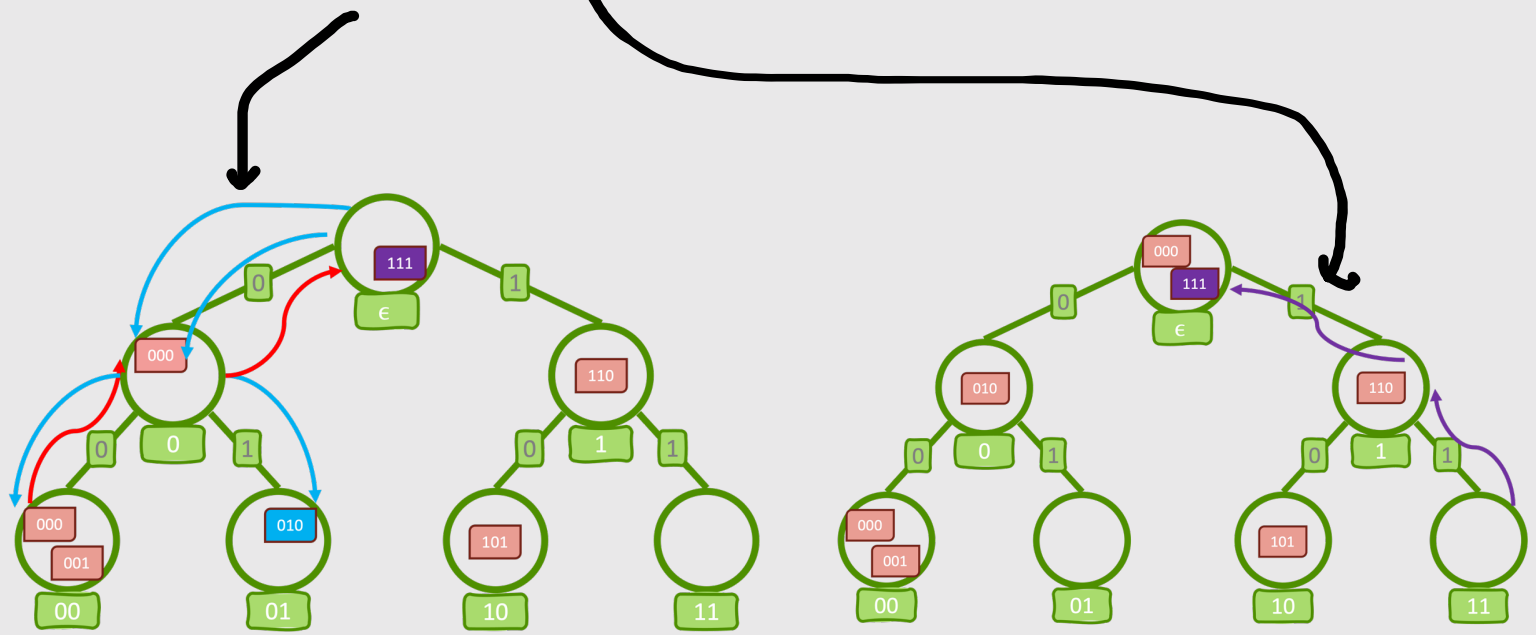
Proof sketch:

- 1 for pull-up
- Fractional occupancy ensures success

  after $\left\lceil \frac{1}{1-f} \right\rceil$ tries, in expectation



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- Property 2:

  Access cost of SeedTree $\leq 2 - \log(f)$ ·Access cost in MRU Tree.

- Most Recently Used (MRU) Tree:

  More recently accessed items ⇒ Lower level in the tree

SeedTree Analysis

- Objective (over time and in expectation):

  SeedTree is dynamically optimal.

- Property 1:

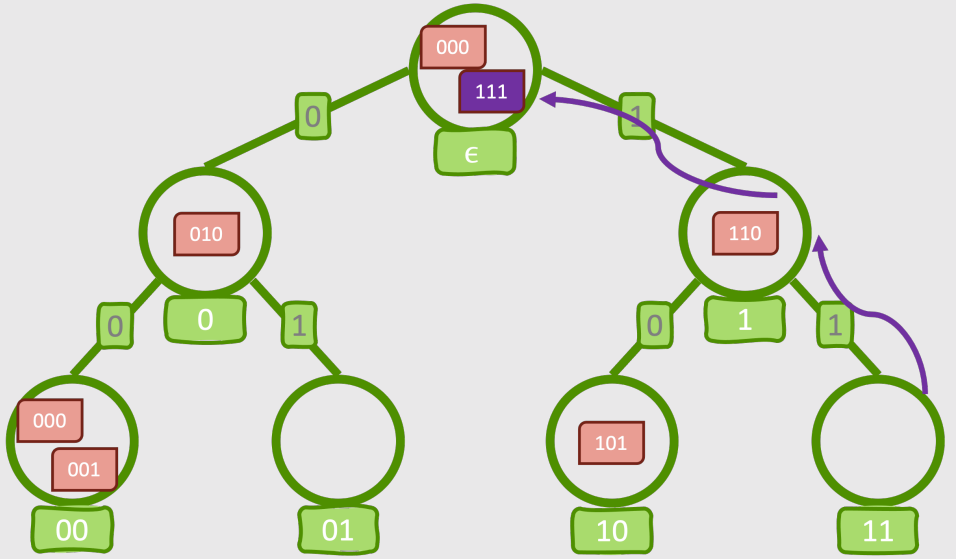
  Reconfiguration cost of SeedTree $\leq 2 \cdot \left(\left\lceil \frac{1}{1-f} \right\rceil + 1 \right)$ ·Access cost of SeedTree.

- Property 2:

  Access cost of SeedTree $\leq 2 - \log(f)$ ·Access cost in MRU Tree.

Proof sketch:

- Recent ones go to the root
- Probability of going a level down decreases exponentially per level



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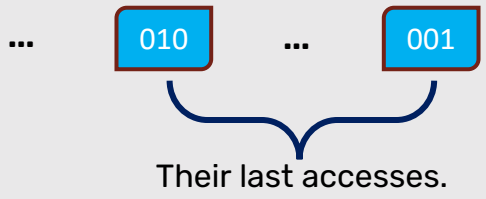
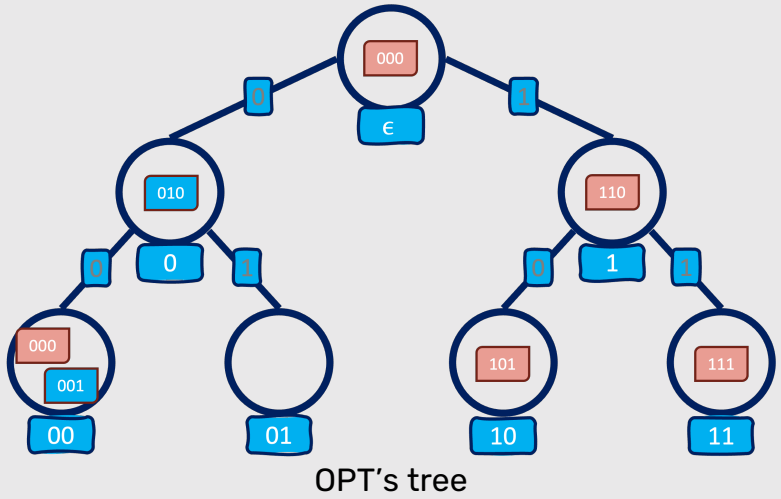
Access cost in MRU Tree $\leq (1 + e)$ ·Access cost of OPT.

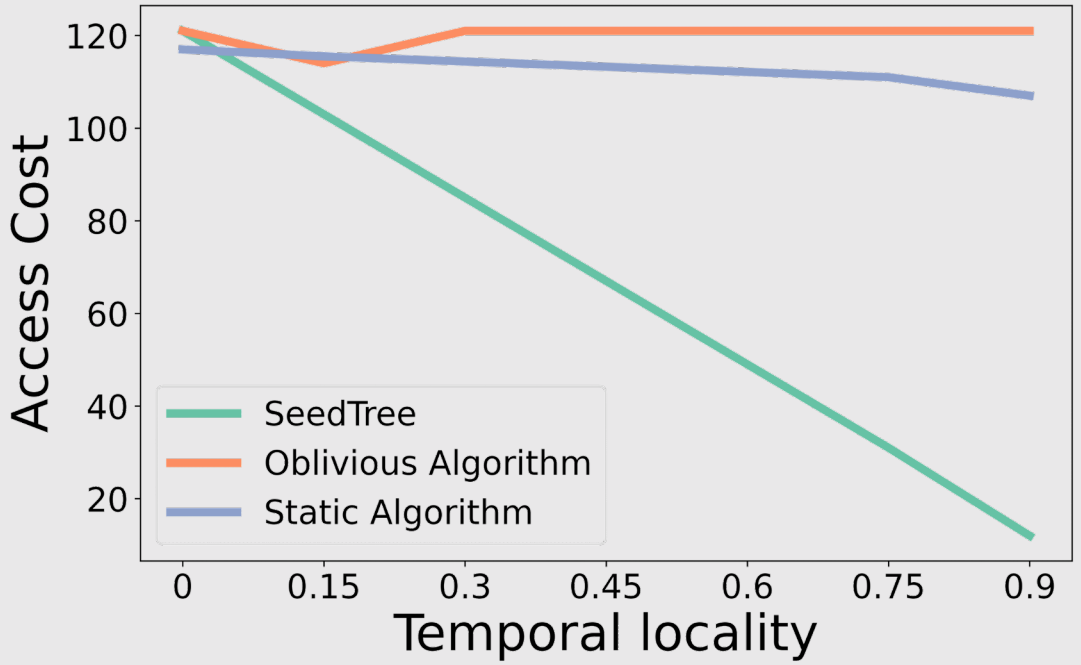
Proof sketch:

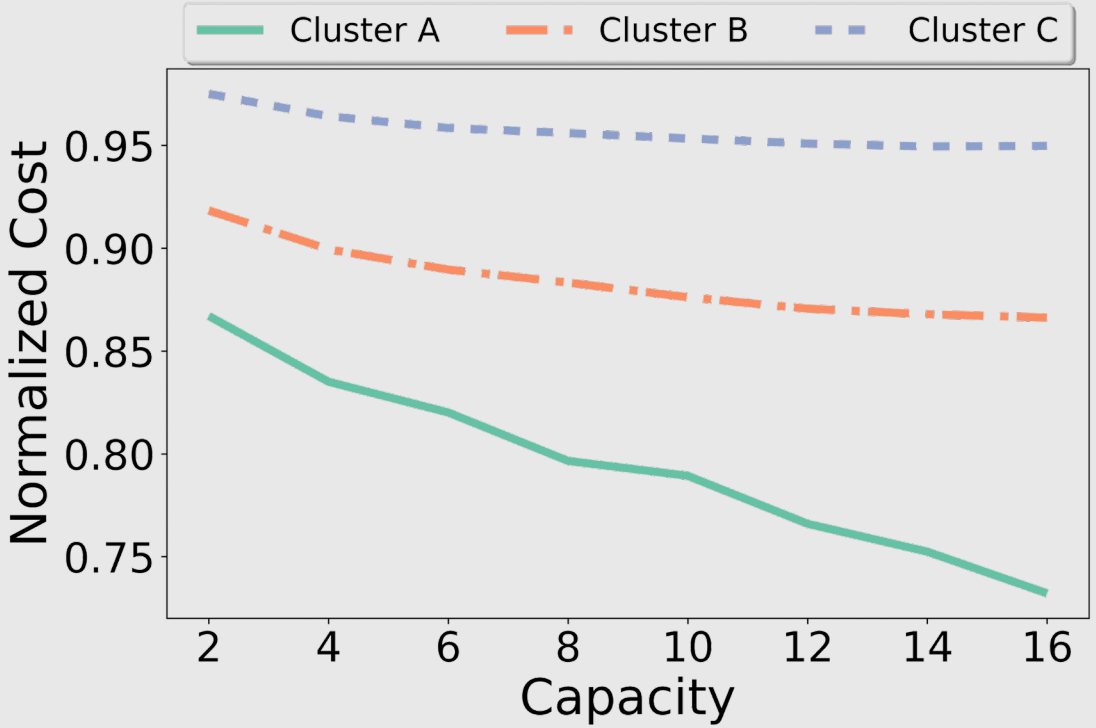
- Potential function analysis based on inversions
- Inversion: item with lower level but accessed earlier





Performance

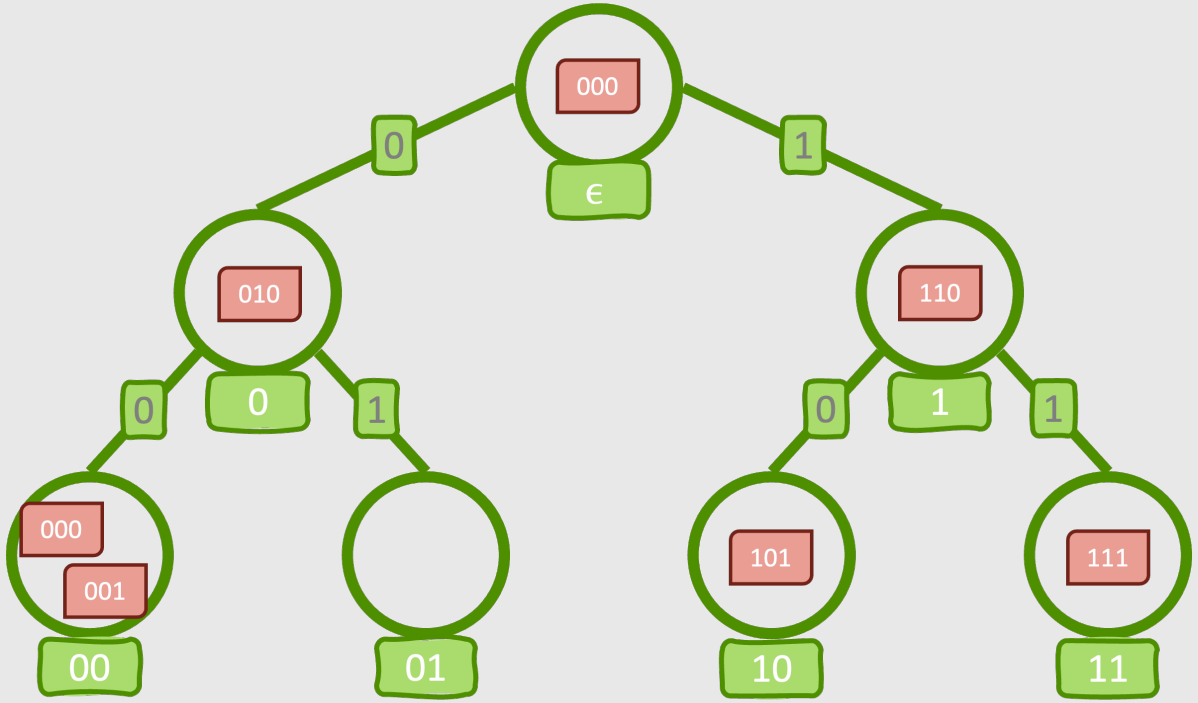




github.com/inet-tub/SeedTree

Conclusion

- Designing a constant competitive algorithm, utilizing randomization in each step.
- Introducing the notion of capacity and item movement for self-adjusting trees.
- Showing significant improvements in the algorithm given inputs with high temporal locality.



Thank You!

Full paper:
arxiv.org/pdf/2301.03074.pdf

Our group's website:
tu.berlin/en/eninet

My website:
pourdamghani.net